Consistent Dynamic Choice, Non-renewable Resource Use, and Uncertainty

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The principles which guide non-renewable resource use are based partly on theoretical investigations of the consequences and the merits of use, which are both uncertain. Existing economic approaches to uncertainty do not correctly reflect a decision-maker's position in time. The power to determine future decisions is overstated, and a limited range of objectives can be investigated.

These problems are addressed by developing a new approach to choice over long time periods. The approach is recursive: each of a sequence of decision-makers decides on the immediate action to take, given the expected consequences, among which are the future actions. Each decision-maker forecasts how future decisions will be made by forecasting what the future decision-makers' objectives and options will be. The resulting forecast actions are consistent: there is no foreseen reason why they will later need revision.

Virtually any sequence of objectives can be investigated with the approach. Applying it to non-renewable resource use over three periods reveals that the optimal initial use: changes if future decision-makers use discount rates different from the first; changes if the future discount rates become uncertain; changes with a change in the time at which future technological improvements become known.

Keywords

economic, decision-making, resource, non-renewable, depletion, dynamic, multi-stage, consistent, recursive, uncertainty, strategy.

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Introduction

Most societies depend on using metals and fossil fuels to maintain living standards. There are limited stocks of these things and they are irreversibly degraded in use. This raises the fear that they may one day run out, causing living standards to collapse. The immediate question is: what implications does this possibility have for **current** decisions?

Attempts to answer this question have greatly refined and clarified the issue. Running out in physical terms is not likely to be a problem, because running out of economically exploitable reserves will occur first. Limited reserves alone are not a problem, but limited reserves for which no substitute can be provided may be a problem. Every refinement of the issue leads to a more detailed restatement of the same questions: what implications for current decisions (on energy use, on exploration, on research) do the future possibilities (uncertain substitutes, uncertain new finds, uncertain future tastes) have?

These questions cannot be answered without making two sorts of judgements, covering firstly what the consequences of current actions are, and secondly how the merits of actions are to be gauged. Because each of these judgements must be to some extent arbitrary, the best situation that a decision-maker can hope for is to be well-informed: to be aware of the range of consequences, and the range of merits, of the options for action, before the decision.

Theoretical investigations play a major part in informing decision-makers about situations which are too complicated for common-sense to be applied. To date, theoretical investigations into the long-term consequences of using metal and fossil fuels have not properly accounted for the interactions between current and future decisions, or for the range of possible attitudes towards uncertainty.

The recursive decision approach developed in later chapters is intended to address these problems, and so contribute to more fully informed decision-making. These chapters are outlined below, after the problems of making decisions about non-renewable resources, in dynamic, uncertain situations, have been more fully discussed.

1.1 Resources and actions

Resources

The set J of 'resources' is defined to consist of all material things seen as presently or potentially useful to humans. This is an explicitly anthropocentric definition. Each resource might improve human wellbeing at some time, because use of the resource enables a preferred situation to be realized, or because continued existence of the resource ensures that undesirable situations are avoided. For example, since coal may at some time be burnt to provide warmth and the ozone layer filters radiation to a comfortable level, both coal and the ozone layer are currently resources. J depends at each time t on the current and expected: preferences of humans; technology (broadly interpreted) with which humans can transform things; environment, or the way things transform themselves.

Each J(t) may include some 'non-renewable' resources $r \in R(t) \subset J(t)$: these are inaugmentable, because their physical quantity $q^r(t)$ can not be supplemented by an environmental process or human actions; each r is irreversibly degraded in use, so that each physical unit can be used only once. Therefore, $q^r(t) \geq q^r(u)$ for all $u > t$, with strict inequality applying if r is in use.

The definition of $R(t)$ is more useful if environmental transformations which supplement or erode $q^r(t)$ at very slow rates, by human standards, are ignored, along with technologies which can supplement or restore $q^r(t)$ at very slow rates relative to the use rate at any current or envisaged t. $R(t)$ depends on the technology, preferences and understanding at t. In principle, all resources are candidates for R, because no state of things can be exactly repeated. If the history of each unit of a resource is important to preferences, then each unit is in principle non-renewable, since each unit has a unique history. However, if preferences make no distinction between unit histories then the resource may or may not be in R.

Use of any $r \in R$ irreversibly alters the opportunities which can be taken up in future. Some future opportunities requiring use of r must be missed, but other future opportunities may be created by current use, so that opportunities overall may expand or contract.

Actions

Actions are all the physical acts undertaken by humans which move or change an object, however small; thinking is an action. Actions and environmental transformations make up the full set of transformation processes, which develop and change each other through time. Many things, including resource stocks, may appear to be untransformed on time scales adopted for human purposes.

There is no single 'correct' set of dimensions for describing transformation processes. For instance, the process with the usual label 'coal mining' is also a physical transfer of mass, a harvesting operation by a community, and a (perhaps) profitable corporate activity. Each description uses only a subset of the dimensions with which the process can be described. Internally consistent sets of dimensions implicitly set boundaries - things outside the boundary are not considered - and occupy a level of aggregation - the transformations described are composed of other transformations which require different, finer dimensions for description.

Strictly speaking only human individuals can act, but it is convenient to refer to group and social undertakings as 'actions'. In this study non-renewable resource actions are described using economic dimensions; actions include 'coal extraction at an opencast mine', and 'total resource use in the time period'. Other consistent sets of dimensions might recognise the actions' place within a 'corporation', a 'community', or an 'ecosystem'.

Some hypotheses about transformations are maintained throughout this study. Environmental transformations are mostly determined by cumulative causation, whereby the nature of things at each instant dictates the nature of things at the next instant. However the transformations may involve a 'random' element, or be poorly understood, so that stochastic relationships provide the best description. By contrast, at least some human actions are **selected** from a feasible set in a way that is not fully determined by cumulative causation. The 'do nothing' or more properly 'just think' action is always among the options.

The concept of an aggregate selection (e.g. society 'chooses' a tax level) is convenient in analysing institutions (e.g. laws, taxes), and in normative discussion of societal options (e.g. which resource use patterns are intergenerationally equitable?).

1.2 Decision-making

There is a sequence of selection, or decision, times. Each decision influences which future transformations and decisions are possible, and which preferences are applied in making future decisions. The influence of each decision extends over the whole future. The distant future impact of the decision may be unimportant to the decision-maker, or highly uncertain, or both.

Decision-makers have two sorts of beliefs: some cover how transformations work, and enable the consequences of possible actions to be explored; others cover preferences or 'the good' and enable an appropriate action to be singled out from the alternatives. Each decision utilizes some beliefs of each sort, although they may be rudimentary and may not be explicitly expressed.

Beliefs about transformations cover the current state of the world, the relationships governing environmental transformations, how others might act currently and in future, and how the decision-maker's future choices might be made. A **set** of actions is believed to be possible in the current, and in each possible future, situation. Scientific theories have immediate application here, in showing (subject to the inductive premise) that certain worlds are not possible.

Preferences, and beliefs as to the good, cover which beings or things should be members of the 'moral community', what rights should be extended to them, which of their interests should be taken into account, how conflicting rights and interests should be dealt with, and how things are to be valued.

Decisions are preceded by various amounts of deliberation. They may be made largely on 'intuition', on general principles, or after detailed consideration of each sort of belief. The deliberation brought to a decision is partly a matter of choice and partly an environmental matter: decision situations are foreseen with varying accuracy. No matter how full the investigation of a decision is, the force of scientific and ethical argument is insufficient to identify an action which is unequivocaly 'best'. One action (perhaps 'do nothing') must be taken, so not all actions can be rejected: if all actions are morally repugnant the dilemma of how to act must still be faced and overcome.

If beliefs about the consequences of action are rational, in that they are based on scientific delineation of the possibilities, then they must contain irreducible uncertainties. Scientific predictions employ the arbitrary inductive premise - that the theory or 'law' will continue to be corroborated in future. Judgement as to the degree-of-belief in rival theories, or rival parameter settings, is arbitrary.

Reality is evolutionary: things which were inconceivable occur, and the result of thought and experimentation cannot be exactly known in advance, so it is known that exact predictions of future beliefs will eventually be wrong. Beliefs are a cognitive structure and so must fall short of the reality they describe on logical grounds, and on the more constraining grounds that human rationality is limited by the capabilities of the human brain.

Other decision-makers may be deciding on action at the same time, so each individual's beliefs must cover the actions of others. If a collective agreement precedes the decision time, beliefs at that time must cover the possibility of defaults. This mutual dependence, or outguessing, introduces a further arbitrary element to beliefs.

The irreducible uncertainties provide an upper limit to knowing the consequences of an action. The limit is reached when beliefs are thoroughly tested for internal consistency using the full set of not-yet-discredited scientific theories of the day. Pragmatically, the available analytical resources do not allow this limit to be approached. The consequences of activities involving non-renewable resources extend to the distant future and so are likely to be particularly uncertain.

Also, choice is inherently normative. A decision-maker who wishes to act in a justifiable way (say, a 'just way' or a 'good way') must first relate actions to consequences and then relate both actions and consequences to 'justice' or 'the good'. There are various competing conceptions of these notions. Some ideas such as 'ethical consistency' throw a great deal of light on the different value positions, but argument cannot fully settle semantic and ethical issues. Non-renewable resource activities are likely *a priori* to influence the existence, nature and well-being of potential persons in the distant future. The ethical arguments about the status of these persons are far from settled at present, so there is irreducible doubt on these grounds (among others) about the merits of non-renewable resource actions.

Uncertainty and the normative nature of choice set limits to the powers of rationality in determining the 'best' action at a decision time, but the alternative decision-making schemes (perhaps religious codes, or randomized selection) are more arbitrary still. Therefore, any investigation can only seek to inform the decision-maker about the meaning and merits of actions. There is no final answer, but 'informed decision-making' is more or less achieved when, before the decision time, the decision-maker is more or less aware of the consequences and merits of each possible action, according to each tenable set of beliefs.

Because analytical resources are limited informed decision-making is a target, and in practice general principles are used to identify for which decisions the decision-maker seeks which information, and to select actions on the basis of existing information. The principles take the form of constraints on actions, or of objectives which are to be used to decide on action, or of less formal 'feelings' which influence actions in these ways. Note that seeking further information is an action. Principles arise in many ways; theoretical investigations play a major role in developing them.

1.3 Current decision-making about non-renewable resource use

Currently, societal decision-making about non-renewable resources is less than fully informed in several obvious respects. Most of these correspond to theoretical deficiencies and are freely admitted to by many decision-makers and theoreticians. Chapters Two and Three enlarge on the following points.

A very narrow range of ethical positions are examined in practice or in theoretical exercises. The prevailing orthodoxy is the ethical judgement of 'consumer sovereignty', whereby the beliefs of individuals acting in markets determine which resource activities are appropriate (within the limits of a complex of environmental, safety, planning and anti-trust law). The beliefs of current buyers and sellers of resources and downstream products (and those who could buy or sell) therefore determine which consequences prevail.

Consumer sovereignty is open to question on several grounds: why are individuals' choices 'good'?; why should market choices prevail?; why should the existing distribution of endowments (power to choose) be respected? When uncertainty is present: why should individuals' expectations and attitudes to risk receive approval? This is not to suggest that consumer sovereignty should be abandoned. However, consumer sovereignty is not sufficiently morally compelling to make redundant the investigation of other ethical positions.

Theoretical examinations of Utilitarian and Rawlsian positions as regards non-renewable resources have been made in optimal growth frameworks. The results are hard to relate to beliefs about the merits of current action. This is in part because a solution in these models is a time path of resource actions, or a complete strategy of resource actions, and not a mapping demonstrating how current actions should vary with current beliefs.

Few sets of assumptions about possible future transformations are used in practice or in theory. In New Zealand government practice is guided largely by theoretical economic results and casual empirical observations. Another guide is econometric forecasting, which is usually extrapolative but may also cover subjective forecasts of future shifts in government policy. These generally have horizons of less than three years.

An exception to this has been the planning studies of the Ministry of Energy which cover fifteen years in general and longer for some issues including, notably, gas reservoir depletion. These studies were undertaken in a rolling planning scheme and used a lot of physical data. The consumer sovereignty judgement was used, with the results generally being plans which maximized the net present value of revenues adjusted to reflect taxation content. Major uncertainties were generally investigated with scenarios.

The most ambitious attempt to inform decision-makers about distant future possibilities also used scenarios and a great deal of information about possible transformations. The "Four Futures" study (Boshier *et al.*, 1986) developed four scenarios of the future 50 years for New Zealand embedded in the wider world. Changes were explicitly related to postulated shifts in technology, political and economic conditions, demographics, and attitudes (ethical positions). Stringent attempts were made to ensure that each scenario was internally consistent at all times. The four scenarios were very different, and it was concluded that this demonstrated a need for 'resilience' in the face of the major uncertainties. The study solidly demonstrated that vastly different futures are consistent with current beliefs. It could not further advise decision-makers, about the meaning and merits of current actions, without adopting one or more value statements and likelihood statements; i.e. without adopting a position within the tenable sets of belief.

Theoretical investigations of resource issues generally examine a highly restricted set of possibilities - most often including perfect foreknowledge. As with ethical positions, the solution sought is not an explicit relationship between beliefs (about possibilities, now) and current actions, but is a time-path or strategy. Many of the beliefs remain implicit in the dynamic structure of the model used, and many could never be tenable belief sets on commonsense grounds. For example, no initial decision-maker could reasonably believe that no future decision-makers will wish to deviate from the strategy adopted at the initial moment under the initial decision-maker's definition of appropriate actions. This belief is implicit in virtually all studies to date.

1.4 Outline of this dissertation

The sections above contend that the dynamic, uncertain context of decision-making about non-renewable resources is poorly understood, or at least poorly incorporated, in theoretical investigations. Resource use decisions are therefore likely to be guided by principles which are deficient, in that they do not take proper account of this context.

It follows that a deeper investigation of non-renewable resource use in dynamic, uncertain contexts has the potential to make decision-makers more fully informed about the consequences and the merits of their actions. This dissertation contributes to such an investigation.

The next two chapters demonstrate that the context is, in fact, poorly understood. The economic approaches to decision-making in dynamic uncertain situations are briefly reviewed in Chapter Two. These approaches do not correctly reflect a decision-maker's position in time, rely on a questionable 'solution' concept, and can only investigate a limited range of attitudes toward uncertain outcomes.

The investigations of non-renewable resource use in dynamic uncertain contexts are reviewed in Chapter Three. The limitations of the economic approaches carry through to these investigations. The chapter concludes with a discussion of the need for an approach which can deal with the choices of a sequence of decision-makers with varying attitudes to uncertainty.

Chapter Four develops such an approach. The recursive decision approach forecasts future choice procedures, treats future chosen actions as part of the forecast, and restricts each decision-maker to implementation of the current actions only. The recursive approach is first developed heuristically by extending a decision tree, and then stated in more formal terms. The existence of a 'solution' and possible extensions of the approach are discussed before the chapter concludes by discussing the potential of the approach.

The recursive approach is elaborated upon in Chapter Five. A simple example brings out the structure of the approach, and choice procedures for use in recursive models are discussed. Analytical tractability is briefly covered before some mappings useful in representing 'solutions' are outlined.

The next three chapters apply the recursive approach to issues in non-renewable resource use, for the dual purpose of testing the approach and investigating the issues. Chapter Six examines the reaction of initial resource use to expected changes in future choice procedures, and Chapter Seven extends the enquiry to uncertain contexts. Chapter Eight investigates the importance for resource use of the timing of uncertainty resolution. Each chapter concludes with a discussion of the findings and of the performance of the approach.

The dissertation is concluded in Chapter Nine. This chapter summarizes the preceding arguments, and evaluates the contribution made by the thesis to understanding of non-renewable resource use in dynamic, uncertain contexts. Finally, some implications for policy, and some areas where future research appears to be worthwhile, are discussed.

Chapter Two

Economics and Uncertainty

Economic approaches predominate in theoretical investigations of non-renewable resource use under uncertainty. To facilitate the review of these investigations the central features of economic approaches to uncertainty are now examined.

The theory of individual choice is discussed before social choice theory is outlined. Some factors which are important in intertemporal contexts are then reviewed. The conclusions identify the main limitations of using the approaches as the basis for investigating appropriate non-renewable resource use. Some methods of investigating choice under uncertainty are illustrated in the Appendix.

2.1 Individual choice under uncertainty

Most economic investigations of uncertainty describe or explain the choices of individuals or firms (Diamond and Rothschild, 1978; Lippman and McCall, 1981). Social choice when outcomes are uncertain has been relatively neglected, and is treated as analogous to individual choice.

Choice under uncertainty involves selecting an action which has a number of possible consequences. Two suppositions underpin the mainstream theory of choice under uncertainty: firstly, a probability measure is used to describe the likelihood of possible consequences; secondly, individuals behave as if they are maximizing the mathematical expectation of a 'utility' indicator defined over the possible consequences (Arrow, 1970; Schoemaker, 1982; Machina, 1987).

2.1.1 The probabilistic supposition

The concept of a probability measure is mathematically clear-cut, but its meaning and use are disputable (Faden, 1984; Schoemaker, 1982).

Objective definitions

Some schools define probability 'objectively': as being external to any observer. The frequency school defines probabilities as the observed or extrapolated long-run likelihoods of the outcomes of repetitive trials. A problem is that **exact** replication seems to call for the same outcome. This concept is inapplicable to unique events, including most (perhaps all) events of economic interest, such as the size of a coal reserve, or future oil prices.

The classical (Laplacean) postulate is that there is an exhaustive set of 'elementary' outcomes. The 'Principle of Insufficient Reason' applies: if there is no reason to believe that any elementary outcome is more likely than the others they are all equiprobable. The standard conjunctive rule gives the probability of any compound outcome. Taking elementary outcomes to be equally probable makes this definition of probability appear circular.

A third objective view is that probability is a weak form of logical statement: a relationship between the truth of a proposition and the weight of evidence in support of it. However, no satisfactory objective measure of 'weight of evidence' has been found.

Subjective definitions

Economic theories of choice usually invoke subjective probabilities. These personal mental properties amount to 'degrees of belief' in the truth of propositions, whether the propositions are about coin-tossing or future oil prices. The primitive concept is 'belief' - it is postulated that any 'coherent' structure of beliefs satisfies axioms which give degrees of belief the same properties (summing to one, conjunctive and disjunctive rules) as a probability measure.

Supplementary postulates about learning, or the modification of beliefs by experience, are necessary to 'explain' why different people attribute the same probabilities to such things as coin-toss outcomes. Coherent sets of degree of belief satisfy the 'laws' relating joint, marginal and conditional probability distributions: Bayes Rule governs envisaged updating of beliefs over time (Savage, 1954; Pratt *et al.*, 1964).

Other positions

'Risky' situations are where the assumptions of the frequency school are met, or the law of large numbers operates, and in other situations intuitive estimates, and intuitive judgements as to the reliability of those estimates, are used in making choices (Knight, 1946). Intuition is not subjective probability - entrepreneurs must contend with unknowable change. Profits are the reward of successful 'uncertainty bearing'. Knight (pxiv) is "...puzzled at the insistence of many writers on treating the uncertainty of result in choice as if it were a gamble on a known mathematical chance...".

An alternative postulate is that agents attach a degree of 'surprise' to each possible outcome. Potential surprise is a mental property measuring (*ex-ante*) how surprised the agent thinks he or she would be if the outcome happened. The surprise of a set of outcomes is the smallest surprise of an outcome in the set, so surprise is not subjective probability: their conjunctive properties are different (Shackle, 1949-50). Surprise is deficient because it does not match probability in repetitive situations (Arrow, 1951b, p419). Shackle and the 'Post-Keynesian' and 'Austrian' schools now treat surprise in a context of 'historical', 'subjective', or 'real' time, where the perceived past and future are properties of the present, and action is a creative process (Shackle, 1972; Ford, 1983; Bausor, 1982-83, 1984; O'Driscoll and Rizzo, 1985).

The behavioural view of uncertainty also emphasizes that in the evolutionary human situation qualitatively new situations are always occurring. In addition the limited reasoning powers of humans provide only bounded rationality, which is insufficient to sustain a consistent probability-based picture of the future (Simon, 1959; 1983).

Psychological research indicates humans do not form consistent probabilistic beliefs, and better corroborates other theories on belief formation (Tversky and Kahneman, 1974; Schoemaker, 1982; Machina, 1987).

2.1.2 The expected utility maximization supposition

The maximization of mathematical expectation has a long history as a basis for choice theories (Arrow, 1951b). Axiomatic versions demonstrate that if agents meet 'rationality' axioms on preferences, then a utility function over consequences exists, such that choices maximize the mathematical expectation of the utility of consequences (von Neumann and Morgenstern, 1944; Savage, 1954; Pratt *et al.*, 1964). This approach is labelled subjective expected utilitarian (SEU) choice.

The utility function is not the classical economic measure of 'pleasures and pains'. It represents the agent's 'attitude to risk' as well as to the prospects. This is illustrated in the Appendix.

The axioms are only met by monotonically increasing utility functions, bounded to prevent existence of gambles of 'infinite' expected utility (Arrow, 1970). If agents are risk-neutral or risk-averse over the entire range of consequences, utility functions are concave (Pratt, 1964). Functions with inflection points are needed to 'explain' the coexistence of gambling and insurance (Friedman and Savage, 1948).

Other positions

Many theories of choice have been proposed. Some have been expounded for financial or income consequences only, although their central criterion may apply equally well to the utility of consequences.

Examples are:

- Stochastic dominance: eliminating inferior (dominated) decisions. A decision is first degree dominated if there is another decision which has better (utility of) consequences under all possible futures, and second degree dominated if another decision has a higher probability of exceeding every possible (utility of) consequence level (Lippman and McCall, 1981; Bawa, 1982).

- Approaches not using probabilities (beyond requiring bounded possible consequences) deal with a situation dubbed 'total ignorance'. Under the maximin criterion the preferred action maximizes the (utility of the) consequences occurring in the worst possible case (Wald, 1950). Maximax is analogously defined, and these are generalized under the Hurwicz criterion where the preferred action maximizes a linear combination of best possible and worst possible (utility of) consequences (Arrow and Hurwicz, 1972). The minimax regret approach postulates that the preferred action minimizes the maximum 'opportunity cost' which could occur, given that action. The Laplacean criterion takes 'elementary' outcomes as equiprobable and seeks the action which maximizes the expected value of the (utility of) outcomes.

Sets of axioms providing foundations for these approaches are discussed in Maskin (1979).

- Mean-variance approaches maximize the mean outcome of the gamble minus a multiple of the variance of the gamble. The size of the multiple supposedly captures the attitude to risk. In general this is not equivalent to expected utility maximization (Markowitz, 1959; Porter, 1973).

- The safety-first approach minimizes the probability of a defined set of bad outcomes (Roy, 1952; Telser, 1955; Arzac, 1976).

- Shacklean surprises of defined absolute losses and gains can be used to form a 'focus loss' and 'focus gain' on the basis of which the agent's preferred actions can be identified (Shackle, 1949-50; Arrow, 1951b). Similarly the probabilistic expected loss may be the focus for choice (Fishburn, 1984).

- The behavioural theory of satisficing postulates that the first action which is found to meet specified minimum standards of consequences is adopted (Simon, 1983).

- A currently active research field is the exploration of the variants of SEU theory given by weakening the axiomatic foundations. 'Prospect theory' (Kahneman and Tversky, 1979), 'regret theory' (Loomes and Sugden, 1982), and 'generalized EU theory' (Machina, 1982), all envisage that individuals have preferences about the probabilities of outcomes, which are not fully captured in forming an expected value measure.

2.2 Social choice under uncertainty

Prescriptive economic investigations of social choice under uncertainty use analogies of the theory of the individual agent.

2.2.1 Description of consequences

According to the individualistic view of societal welfare, the socially relevant consequences of an action are (in principle) the resulting flows of commodities and experiences of all individuals. With uncertainty, a (possibly infinite) set of fully described flows represents the set of possible consequences of an action.

It is not clear whether the possible *ex-post* effects, or the *ex-ante* effects including individuals' experiences of uncertainty, are more appropriate as the basis for social choice (Diamond, 1967; Dreze, 1970; Starr, 1973; Guesnerie and Montbrial, 1974; Mirlees, 1974; Harris and Olewiler, 1979; Hammond, 1981). This is further discussed below.

Probabilities over the *ex-post* possibilities cannot be objectively based, but it is not clear whose subjective probabilities should be used as the basis for *ex-post* social choice. Expert opinion is often judged to be appropriate, but like any other belief, expert opinion can only ever be tested (before the fact) for its coherence.

2.2.2 Social welfare criteria

It is well known that with or without uncertainty, constructing a social welfare function from individual preferences requires judgemental input: social and individual spheres of preference must be specified; the Arrow Impossibility results must be circumvented; if individuals' utilities are the basis, the measurability and comparability of utility must be established (Arrow, 1951a).

Further, different actions lead to future people having different natures, and eventually being genetically different people. The social welfare judgements therefore cover choices between different existences, and not only different consumption or utility levels for the same people. Unsettled ethical issues are involved here (Parfit, 1982).

A social criterion covering uncertainty requires in addition an assessment of the relevance of the uncertainty experienced by individuals, and perhaps a statement on the appropriate social attitude to (aggregate) uncertainty. There are two major positions:

Ex-ante

Here, individuals' initial experience of uncertainty is judged relevant. The individual's preference order or expected utility covers the individual's view of their whole future. Aggregating initial expected utility over individuals to form a measure of social welfare includes the expectation in the 'mental' welfare of the individual.

Ex-post

Here individuals' experience of uncertainty does not influence the social evaluation. Individual preference orderings or utility functions over 'delivered' *ex-post* outcomes are the basis for aggregation. The existence of such orderings in an uncertain world, and their relation to the expected utility orderings, are open questions.

It is not clear that maximizing the **expected** value of an *ex-post* measure is appropriate. A 'risk-averse' approach might maximize the minimum possible level of the social welfare measure, perhaps because gambles here are not 'affordable'. There is some ethical disagreement on this issue (Rawls, 1971, 1974; Harsanyi, 1975).

Comparison

The merits of the *ex-ante* and *ex-post* positions are the subject of debate (Harris and Olewiler, 1979; Hammond, 1981). The *ex-ante* position is generally adopted in extending the 'efficiency' (Pareto Optimality) criterion to uncertain situations.

The welfare theorems can be extended to show that an intertemporal equilibrium under uncertainty is *ex-ante* efficient, and that *ex-ante* efficient equilibria can be supported by a price system under certain conditions. These theorems use much weaker axioms on choice than those supporting SEU choice (Debreu, 1959).

However, *ex-ante* efficiency is questionable on several grounds. It takes agents' likelihood judgements as parameters whatever they may be. These judgements seem questionable in ways tastes are not: should beliefs which are demonstrably irrational count in determining the social allocation?; are individuals' choices of **how** informed to be, to be respected? Similarly, agents attitudes to risk are parameters, and are questionable: must society respect the wishes of (say) extremely risk-loving individuals?

If equity is considered on the basis of *ex-ante* judgements, further issues arise. Individuals with pessimistic beliefs (Hey, 1984) must be given correspondingly high weight.

Ex-post efficiency can be defined over the full set of agents, by event (Starr, 1973). A less demanding criterion is the "Allais optimum": the expected value, over events, of a function of agents' *ex-post* utilities (Mirlees, 1974). The latter makes use of the questionable 'social probabilities' referred to above.

Neither *ex-post* criterion can in general be achieved by a decentralized price system with transfers. The exception is when the *ex-post* social welfare function is linear in individuals' *ex-post* utility, and 'social' probabilities coincide with all individual probabilities. In general, the transfer mechanisms which seek to support the *ex-post* target create the potential for *ex-ante* gains from trade, and, if acted on, these prevent the *ex-post* target from being reached (Harris and Olewiler, 1979).

Intertemporal inconsistency (see below) is a possibility for both *ex-ante* and *ex-post* approaches to social choice. The *ex-post* approach can be limited to using consistent social welfare judgements or preferences, but this does not alter the *ex-post* implementation problems.

Aggregate models

Directly aggregated models also require extra judgements to cover uncertainty. Optimal growth models generally allow for uncertainty by maximizing the expected value of the original certain objective, which is often the sum over time of discounted utilities.

This is most easily interpreted as an *ex-post* position: the expected value of a measure of 'delivered' social satisfaction is maximized. The questions applying to *ex-post* social welfare functions apply here also: whose probability judgements are used, and why is the neutral attitude to aggregate risk appropriate?

2.3 Uncertainty over time

The approaches above do not distinguish between 'static' and 'dynamic' uncertain situations. In the former the chosen action is immediately followed by resolution of all uncertainty as the consequences become known. In the latter the chosen action is followed by a sequence of resolutions over time, perhaps with intervening further actions.

2.3.1 Solution concepts for intertemporal models

'Dynamic' or intertemporal models cover sequences of time periods. The concerns of decision-makers are likely to extend beyond the immediate period. Decision-makers preferences will likely distinguish between future actions as well as current actions.

Some terminology is required to identify types of intertemporal sequences. An arbitrary 'plan' is a sequence of actions, one for each time period. An arbitrary 'strategy' is a collection of actions, one for each possible state of the system in each time period. 'Policy' is sometimes used for 'strategy', particularly where the strategy comprises a time-invariant function from the state to the optimal action. A **precommitment** plan or strategy is a plan or strategy discussed or selected by the initial decision-maker as if **all** decisions were within the power of the **initial** (version of the) decision-maker.

A 'commitment' is an action a decision-maker can irreversibly adopt. A commitment extends from the moment of adoption until the first time at which the action can be reassessed, and changed if then desired. A 'recourse' is a future action which may be resorted to if some specified outcome happens. A 'forecast' is a consistent belief structure covering all future actions and outcomes, perhaps represented by a probability tree or a stochastic process.

2.3.2 Intertemporal consistency

Economic approaches to intertemporal choice under uncertainty generally examine one decision-maker's choice of precommitment strategy. This strategy is intertemporally inconsistent if foreseeable changes to preferences lead to revision of the strategy (Strotz, 1955-56; Pollak, 1968). That is, it is foreseen that at some time an action which was initially discarded will come to be preferred and adopted. This might occur because a later (version of the) decision-maker, with different preferences on later actions, makes later decisions.

A sufficient condition, for an optimal precommitment strategy to be intertemporally consistent, is that all **preferences** are consistent. That is, later choices are the same as those preferred by early decision-makers.

Only a **time sequence** of preference positions can be intertemporally (in)consistent, but it is sometimes stated that a single preference structure is intertemporally (in)consistent. What is meant is that using the preference structure in each period, over outcomes dated relative to that period, eventually leads to (in)consistent choices. Non-exponential discounting procedures, and risk attitudes which may vary with the time until the risk, are possible bases for preferences which are intertemporally inconsistent.

Two approaches have been considered for intertemporal choice when preferences are inconsistent. Neither explicitly allows for uncertainty. The first approach, 'naive' choice, consists of ignoring the inconsistency and repeatedly implementing the initial action of the optimal precommitment plan. Clearly, a 'naive' decision-maker adopts a plan despite knowing it will not be followed, so 'naive' choice does not seem rational.

The second approach, 'sophisticated' choice, involves selecting the best plan according to initial preferences, from among the plans which will be carried out. This approach seems more rational, but can present analytical difficulties, as decision-makers may well face non-convex, and perhaps open, sets of implementable plans (Peleg and Yaari, 1973).

The approach has also been criticized as amounting to "a desire for inflexibility", because it need not favour keeping future options open

(Kreps, 1979). Hammond (1976) considers sophisticated choice is a questionable way of achieving consistency, because the resulting choice mapping may be 'incoherent'. This means that addition of options can lead to a switch in the chosen plan **within** the original option set, implying that sophisticated choice is not equivalent to an order, or to maximization of a utility function, on the intertemporal plans. This does not seem to be a compelling criticism, because the full set of plans is not the option set for any decision-maker with a place in time.

2.3.3 Multi-period SEU choice

When there is uncertainty, the multi-period extension of the SEU approach (MPSEU) deals with choice between precommitment strategies, by maximizing the expected value of a utility function defined on the time-streams of outcomes. Whether the strategy is intertemporally consistent depends on whether the future objectives are consistent with the first.

MPSEU takes no account of the timing of uncertainty resolution. Each strategy leads to a set of outcome possibilities, which is gradually reduced with time. MPSEU uses the outright probability of each member of the set, but ignores the structure of its resolution over time.

A number of approaches seek to generalize MPSEU. Choice procedures can be constructed so as to separately reflect time and risk preferences (Selden, 1978), or to reflect a preference for flexibility (Koopmans, 1964; Jones and Ostroy, 1984).

Since the SEU rationality axioms apply to static choice under uncertainty, they also apply to the **initial** choice in any multi-stage context. Therefore, if the axioms are accepted, a utility function defined on the **whole** forecast future (following each initial event) exists, such that initial choices maximize the expected value of this function. Many non-

MPSEU choice procedures are therefore in full agreement with the SEU axioms.

2.3.4 Social risk-aversion over time

In aggregate formulations, the convention is to label society 'risk neutral' when uncertainty is accounted for by employing the **expected value** of the measure of delivered social welfare. The linear function in Figure 2.1 depicts this social risk attitude: the expected value of social welfare is not affected by the transformation. By contrast society is labelled 'risk averse' when social welfare is taken to be an increasing, strictly concave function of the integral measure. Here, as the concave function in Figure 2.1 shows, the risk attitude lowers the expected value of the welfare measure whenever uncertainty is present. This is further illustrated in the Appendix.

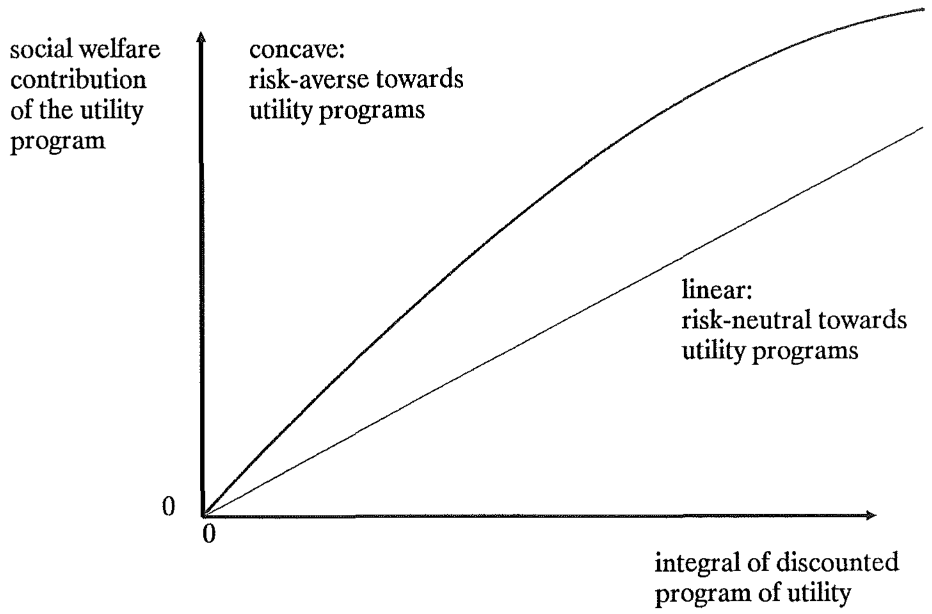

Figure 2.1: Social risk attitudes

There is a need, when using utilitarian objectives, to distinguish between types of risk aversion. The standard integral of discounted concave instantaneous utilities is a risk-averse utility indicator with regard to **each time's** outcome: the integral is concave with respect to each time's utility **argument**. Similarly, for stationary utility and stationary profiles of utility argument, the integral is concave in the **profile level** and in this sense is risk-averse over outcome sequences: a mean-preserving spread in the uncertainty about the profile level will decrease the expected value of the integral.

However, the expected utilitarian form exhibits no risk-aversion about total discounted **utility** levels: the overall objective is linear in this level, and hence risk neutral towards it. In this study, this property is referred to as 'program risk-neutrality'. When it appears, a mean-preserving-spread in the possible integral values leaves the overall measure of worth unchanged.

2.4 Conclusions

The economic approaches to choice under uncertainty are questionable on many grounds. Investigations of non-renewable resource use which employ these approaches are equally questionable.

The model of individual choice is not well-developed for multi-period situations and is at odds with empirical evidence from static situations. Much of the richness of attitudes, towards risks of different sizes at different times, cannot be explored with the standard MPSEU approach. No model yet allows that preferences change with time in an uncertain way: the position in time of decision-making is poorly reflected.

These limitations apply equally strongly to the economic approaches to social choice under uncertainty. Social choice is likely to require consideration of long time periods, so the largely static nature of the theory is a serious weakness. Long time periods might be expected to require consideration of changing preferences, and of complex attitudes to unfolding possibilities. Neither of these can currently be explored: *ex-ante* approaches deal with individuals' 'once and for all' selections, and both *ex-post* and aggregate approaches employ social objectives of MPSEU type, which ignore the timing of uncertainty resolution.

Ethical questions about the two approaches to aggregation (*ex-ante* cf. *ex-post*), and about how to treat potential persons, remain unresolved. This weakens the recommendatory power of investigations using the approaches.

Despite the limitations of the economic approaches, there is currently no alternative which offers a more complete way to explore resource use issues with long-run uncertain consequences.

Chapter Three

Depletion Under Uncertainty: The Economic Literature

Economic approaches predominate in theoretical investigations of non-renewable resource use under uncertainty. These studies are reviewed below, so as to establish their current potential for informing decision-makers.

The theory is grouped firstly by approach, and within this largely by what is uncertain. In all cases the studies build on investigations of a fully known situation; that is, uncertainty is always treated as an 'additional complication'.

The first group, the descriptive or 'positive' examinations, are relevant to optimal societal resource use for two reasons. Firstly, an understanding of how resource allocation systems operate is an important prerequisite to achieving many societal goals. Secondly, the societal goal may be *ex-ante* efficiency, or, almost equivalently, consumer sovereignty may be deemed to be appropriate. It is well-known that these standards are often satisfied by the decentralized systems studied in 'positive' approaches.

The investigations in the second group are directly normative. They use a modified optimal growth theory approach: an aggregate technology and resource base limit the consumption possibilities, and an explicit societal objective is specified, so that optimal resource use patterns can be derived.

The conclusions assess the literature as a whole, and identify the chief limitations of the theory to date.

3.1 Early views on depletion and uncertainty

Prior to 1974, prescriptive economic investigations of non-renewable resources use deterministic approaches. The view commonly adopted in early 'positive' work is that uncertainty about future prospects compels producers of non-renewable resources to speed-up early extraction (Paish, 1938; Watkins, 1944).

One pathbreaking work briefly mentions uncertainty in connection with a social optimum based on consumer surplus. In questioning whether the interest rate should be used as a discount rate, it is claimed that the view that "...future pleasures are ethically equivalent to present pleasures of the same intensity..." must be tempered with the facts that capital is productive (a partial equilibrium framework is used), **and future pleasures are uncertain** (Hotelling, 1931, p145). The nature of this uncertainty is not further explored, and there is no indication of which of the possible "pleasures" should be evaluated, or if the appropriate discount rate varies with the risk.

If the social objective is to maximize net present revenue from the resource, risk and uncertainty can be taken into account as deductions from revenues and additions to costs, according to Ciriacy-Wantrup (1944). This adjustment is thought to make it possible to treat resource use plans as finite, since the additions to costs and deductions from revenues at some distant date make use after that date appear worthless.

A third view is that projections of the distant future become so nebulous that the best approach is to assume the net benefit function does not change after "...some finite year in the future after which changes are too vague to project..." (Burt and Cummings, 1970, p589). The uncertainty before and after this finite year is to be treated by using the best deterministic approximation available.

These three views account for uncertainty by adjusting the deterministic evaluation procedure. In each case a single specified outcome is evaluated. There is no explicit examination of strategy, or foreseen need to adjust plans given new information.

3.2 Descriptive investigations

Until recently 'positive' economic investigations assumed resource owners and consumers had sufficient knowledge of the future to establish and follow an intertemporal equilibrium with no need for strategy (Hotelling, 1931; Herfindahl, 1967; Peterson and Fisher, 1977; Crabbe, 1977; Dasgupta and Heal, 1979; Withagen, 1981).

Uncertainty weakens the validity of the 'partial equilibrium' assumption, which is relied on in considering the resource sector in isolation. Risky extraction decisions depend on which other risky activities are available. A portfolio of activities may be adopted to reduce risk. Agents may share risks or obtain insurance. These possibilities affect the appropriate risk-attitude for a resource extracting company.

3.2.1 Expectations: the base case

A fully known, or perfect, expectation structure underpins the deterministic partial equilibrium results, such as the well-known Hotelling's rule (that the equilibrium net price, or rent, of a constant cost non-renewable resource, rises at the rate of interest over time (Gordon, 1967; Levhari and Liviatan, 1977; Devarajan and Fisher, 1981)). The perfect expectations cover all future resource demand functions, the resource quantity and extraction cost, interest rates, and the continuance of the competitive situation including the resource ownership rights.

It is not necessary that **all** agents have perfect expectations. A full set of futures markets for the resource enables information to be shared and the whole equilibrium path to be established at the initial time (Kemp and Long, 1980). Without futures markets, to sustain the intertemporal equilibrium some agents must have sufficient foresight to be able to arbitrage across the spot markets for the resource. This is true if resource owners have perfect foresight of the demand functions, or if consumers know future demand functions and can store the resource costlessly (Weinstein and Zeckhauser, 1975).

With non-renewable resources **myopic** perfect foresight is not sufficient to sustain the **full** perfect foresight equilibrium. Infinite foresight, or a set of futures markets covering an infinite future, is required if the initial price level is to allow the perfect foresight terminal condition (stock exhaustion as choke price is reached) to be satisfied (Stiglitz, 1974).

3.2.2 Competitive extraction under uncertainty

The investigations of intertemporal equilibrium resource use when there is uncertainty are now reviewed. They are grouped according to which item is treated as uncertain.

Price uncertainty

When there is no current price uncertainty, but future prices follow a stochastic process so price expectations increase in variance over time, equilibrium resource patterns can be found by discrete (Weinstein and Zeckhauser, 1975) or continuous (Pindyck, 1980, 1981) stochastic dynamic programming approaches. The extractor adjusts output to the current situation as prices are revealed. The extractor's attitude to program risk may be captured in a utility function over the asset value of the resource stock.

When extraction is costless and owners are program risk-neutral, the expected price rises at the interest rate. Extraction falls over time on average. Under risk-aversion the increasing uncertainty reduces the contribution to expected utility of later extraction, so early extraction increases, and the equilibrium rate of increase in the expected resource price exceeds the interest rate.

If costs are convex in the extraction rate, an increase in price uncertainty causes the cost increases in high price periods to outweigh the cost decreases in low price periods, and induces risk-neutral owners to increase early use. Concave costs reverse the effect.

Alternatively, there may be current **and** future price uncertainty. Extraction is decided each period before price is revealed, and the probability distribution over prices is a stationary function of aggregate supply (Lewis, 1977; Burness, 1978). Owners are now assumed to maximize the expected sum over time of the discounted utility of net profit, with the utility function reflecting risk attitudes.

This reformulation of price uncertainty reverses the impact of risk-aversion found above. The price 'risk' increases in proportion with the extraction quantity, so risk-averse owners reduce initial resource extraction by comparison with the risk-neutral rates. The heuristic procedure of allowing for risk, by increasing the discount rate applied to the expected net profit stream, in this case produces faster extraction and exposure to **more** risk.

Tenure uncertainty

At equilibrium, risk-neutral owners who attribute some likelihood to future nationalization of their resource holdings extract faster than those who perceive secure tenure (Long, 1975). Risk-aversion increases the tendency to faster extraction, but expected compensation can counterbalance the result.

Substitute development date uncertainty

The possibility of substitute development speeds the extraction of risk-neutral and risk-averse owners. The impact of uncertainty about the development date is ambiguous (Dasgupta and Stiglitz, 1976, 1981). An increase in this uncertainty can lead to higher initial extraction rates when the stock is small and lower initial extraction rates when the stock is big, if the (stationary) demand function is elastic enough at high prices. This is because very high rents are immediately attainable when there is a small stock, and these are foregone if and when the substitute appears.

Resource uncertainty

Most investigations of uncertainty about resource holdings use formulations involving maximization of a **societal** utility function, with no mention of markets or demands (Cropper, 1976; Kemp, 1976, 1977; Gilbert, 1979; Hoel, 1978a; Loury, 1978; Heal, 1979). These investigations are reviewed in section 3.3. If the utility function is interpreted as a concave utility of profit function, analogies give some insight into extraction behaviour.

When there is no warning of impending exhaustion, risk-neutral owners slow use in general and risk-averse owners slow use even more under reserve uncertainty. The extraction pattern adopted depends strongly on the probability distribution over reserves. Increasing the discount rate may result in increased exposure to risk.

Different results are found if there is no **current** reserve uncertainty, but future reserves fluctuate according to a stochastic process, so ultimate reserves are unknown. The terminal extraction time is when average profit first falls to zero - perhaps because of lack of reserves - and is unknown initially. With risk-neutral resource owners the expected rate of change of the equilibrium price is the same as under certainty, if extraction costs are linear in reserves. The extraction **levels** adopted by risk-neutral owners may be affected through the impact of uncertainty on the terminal condition. Risk-averse owners speed extraction (Pindyck, 1980).

Exploration

The equilibrium for the certain case provides a basis for comparison. When the increase in reserves given by 'exploratory' effort is fully known, the cost savings from postponing exploration are traded off against the cost savings in extraction which are assumed to follow from maintaining large reserves by exploration.

If initial reserves are small, equilibrium extraction initially increases as reserves are developed by exploration and hold down extraction costs. Eventually, as the returns to exploration worsen, reserves fall and extraction decreases over time (Pindyck, 1978). The associated price path is U-shaped. Large initial reserves lead to a monotonically decreasing extraction rate equilibrium paralleling the Hotelling rule result.

The 'rule of capture' aspect of exploration, whereby resource property rights are given to the finder of the resource, has been examined in a simulation approach (Peterson, 1978). Free entry to homogeneous unexplored territories leads to more exploration, a higher reserve-extraction ratio, and faster extraction, than for the surplus maximizing case. Monopolistic rights to exploration territories lead to the opposite effect.

Uncertainty about exploration results can lead to an increase **or** decrease in both exploration efforts and initial extraction at equilibrium (Devarajan and Fisher, 1982). Under risk-neutrality, and current uncertainty about exploration results, expected marginal discovery cost exceeds rent at equilibrium if extraction productivity is increasing in reserves. The competitive equilibrium is not efficient if the resource industry as a whole faces a risk, due to the scarcity of exploration prospects, which is overlooked by firms in deciding on exploration levels.

With no **current** uncertainty about discoveries, and risk-neutral resource owners, uncertainty has no effect on the expected rate of change in the resource price. Exploratory efforts, extraction rates, and the price **level** at equilibrium may change. The direction and size of these effects is sensitive to the discovery function (Pindyck, 1980).

3.2.3 Uncertainty and market structure

Under uncertainty about the development date of a resource substitute, imperfect market structures conserve resource stocks by comparison with the competitive case (Dasgupta and Stiglitz, 1976, 1981; Stiglitz and Dasgupta, 1981). The market power prevailing over the substitute is important: the prospect of a resource/substitute duopoly may lead a resource monopoly to extreme conservation. Therefore, an attempt to introduce competition to a monopolised non-renewable resource market by creating substitutes may not have the 'natural' outcome of lowering prices and increasing output. Strategic behaviour by the resource owners (delaying) and prospective substitute suppliers (bringing forward) is important in determining the substitute development date and the non-renewable extraction patterns, whether or not uncertainty is present (Dasgupta *et al.*, 1982, 1983).

When the returns to exploration are uncertain, risk-neutral firms with market power in non-renewable resources explore less than competitive firms, since the discovery of extra reserves depresses future resource prices (Solow, 1977; Stewart, 1979). However, 'rule of capture' may alter this result if there is free entry to exploration prospects. The firm may over-explore to protect its monopoly in the resource, and by maintaining high resource prices may induce over-exploration by others.

3.2.4 Expectations and disequilibrium

In the formulations above, agents' beliefs about uncertain elements are modelled as exogenous stochastic processes which the elements are 'known' to follow. These processes are not explicitly related to hypotheses about agents' observations (e.g. of the extraction rate), or to theories the agent accepts (e.g. that prices will support an intertemporal equilibrium), or to empirical observations of belief formation. The stochastic processes are not learnt about as the agent gains experience.

The exogenous stochastic processes are therefore arbitrary as theoretical foundations. They are misleading if real expectations formation cannot support the results. This is more critical when, as in reality, there are few futures and contingent markets in which expectations can be expressed and developed.

The formulations assume instantaneous adjustment to equilibrium - no lags, and no disequilibrium trading. If the results are not reasonably stable under real-time adjustment, then they are misleading for the real world. Investigations of these issues, in depletion context, specify an expectational and lag structure, and then derive simulated dynamic outcomes for comparison with the equilibrium perfect-foresight path.

In three different **adaptive** (i.e. extrapolative) expectations formulations the short run price path is sensitive to the elasticity of demand and the parameters of the expectations form. The paths are unstable; a small perturbation initiates an increasing cycle of fluctuations, or an increasing trend away from a 'well-behaved' path. The resource markets are more stable when owners' expectations are influenced by quantity signals such as consumption rates and resource stocks, rather than purely by past prices (Heal, 1975, 1981).

More sophisticated adaptive expectations formulations lead competitive markets for non-renewable resources to a **constant-price** equilibrium in the short run. These paths do not even asymptotically obey Hotelling's rule. Therefore, the perfect foresight paths cannot be supported, even in the short-run, by adaptive expectations (Marks and Sweeny, 1982).

'Rational' expectations about non-renewable resource use are largely unexplored to date. In this context resource owners must decide about extraction rates and about holding resources or other assets. If owners have rational expectations they foresee and take account of the impact of these decisions on the intertemporal price path.

A unique intertemporal equilibrium exists for the competitive case with non-renewable resources, spot markets, an infinite horizon, demand uncertainty, and risk-neutral resource owners with rational expectations (Orosel, 1985). At equilibrium the resource price paths are expected to increase over time at the discount rate. Prices therefore satisfy a stochastic version of Hotelling's rule.

The price at each time is largely determined by the rational expectations of resource suppliers, so shocks do not have much influence, even though there are only spot markets. As Orosel states: "It may be that actual short run demand fluctuations have too much influence on the oil price and that this price fluctuates 'too much'....to be compatible with rational expectations, in the same way as stock prices move 'too much'... In fact, the importance of rational expectations may lie in their showing the basic principles of economic theory are difficult to reconcile with observations."(p712).

However, the resource price movement which might be expected in a world of cartel members, fringe suppliers, and consumers, all of whom have rational expectations about that price movement, has yet to be determined. Empirical research into expectations forms is required before the impact of uncertainty on non-renewable resource supply can be determined with any confidence.

3.2.5 Ex-ante efficiency and market depletion

The two fundamental welfare theorems relate competitive equilibria to Pareto optimal social states, i.e. *ex-ante* efficient outcomes (Koopmans, 1957; Debreu, 1959). Section 2.2.2 discusses *ex-ante* efficiency as a social goal, but there are many well-known reasons why market outcomes may not be efficient. Many of these reasons are relevant to non-renewable resources, and may not be amenable to solution by (say) government involvement.

Market inefficiencies

Property right structures, market concentration, and the policy environment are all possible sources of inefficiency which are widely discussed in the resource economics literature (Herfindahl and Kneese, 1974; Howe, 1979; Fisher, 1981; Hartwick and Olewiler, 1986).

Private and social risks

If agents face uninsurable risks which have no equivalent at the social level, then market outcomes are inefficient. For example, a perceived risk of nationalization of resource holdings induces inefficient market outcomes (Long, 1975). The reverse is that society faces a risk about the **total** resource endowment, and if this risk is ignored by firms the market exploration levels are sources of inefficiency (Devarajan and Fisher, 1982).

Risk attitudes and uncertainty

The efficiency of market outcomes under uncertainty when resource owners are risk-averse is a subject of dispute. Weinstein and Zeckhauser (1975), Heal (1975), and Hoel (1978a) claim that risk-averse firms' reaction to uncertainty is a source of inefficiency. Kemp and Long (1980) counterclaim that this result follows from inconsistent assumptions about shareholders and consumers, and that the competitive outcomes are efficient. Kemp and Long also claim that a competitive situation and resource base uncertainty are mutually contradictory, because owners must anticipate the possibility of monopoly and this influences extraction.

Availability of markets and uncertainty

When the probability an agent attributes to possible states-of-the-world depends on the agent's actions the Arrow-Debreu proof of efficiency of a full set of contingent commodity markets does not apply (Debreu, 1959). Extraction from uncertain resource stocks involves such dependence. To ensure efficiency the usual contingent markets must be augmented by a set of contingent markets in the uncertain resource stocks *in situ* (Kemp and Long, 1984).

Private and social discounting

There is disagreement about which discounting procedures must be used by individuals, firms, and the state, if efficient outcomes are to be achieved (Lind *et al.*, 1982).

When there is no public sector, and given the usual assumptions (endowments, preferences, technology, enough foresight and/or markets, and contract enforcement) a competitive equilibrium over intertemporal markets brings about equality between: each individual's consumption rate of interest (CRI), the social rate of time preference (SRTP), the market interest rate, the marginal rate of return to private sector investment, and the social opportunity cost of capital (SOC) at each time.

Taxation of company income is a widely discussed reason for a 'wedge' between the SRTP and the SOC (generally it is thought that SRTP < SOC): firms only undertake investments with a pre-tax marginal rate of return that is higher than the CRI by an amount which covers company income tax and the tax paid by the individual shareholder on dividends. "Since consumers and firms are facing different rates of return, there is presumptive evidence of inefficiency..." (Arrow, 1982, p117). The wedge causes a bias against long-term investments, and a lower level of investment. Where there are such taxes, therefore, market outcomes may not be efficient - non-renewable resource use may be too fast.

3.3 Prescriptive investigations of substitute uncertainty

The current desirability of non-renewable resource use is affected by the future availability of substitutes. In any period this availability can be summarized in an opportunity cost curve, analogous to a supply curve.

The future locations of this curve are uncertain, and depend on the preceding success at invention and innovation, resulting from research and development efforts. These vary with the perceived rewards, which are likely to increase as resource use rates, and the remaining stock, fall. Therefore, in reality, choice of resource use rates both affects and is affected by estimated future substitute availability.

Only the impact of substitute uncertainty on resource use is investigated in the prescriptive economic literature. As Table 3.1 summarizes, uncertainty about substitute availability is approximated by an uncertain amount available, or production cost, or invention date.

Table 3.1: Investigations of uncertainty about substitutes

Uncertain item	Uncertain attribute	Type of uncertainty
Future Substitutes	size of flow	exogenous
	production cost	exogenous
	existence	exogenous
	date available	exogenous influenced

3.3.1 The nature of substitutes

Optimal precommitment strategies for non-renewable resource use have been derived for several situations where uncertainty about a substitute is fully resolved at a time T.

3.3.1.1 Uncertain substitute 'costs'

An exact substitute for a non-renewable resource becomes available, without limit, at a fixed 'cost' h, at date T. Only a probability distribution over possible cost levels is available before T (Hoel, 1978b).

Letting t index time, the society must decide on the rate a(t) at which the resource is extracted from the stock Q(t) before T, and the extraction rate b(t) and substitute use rate m(t) after T. The resource and substitute use rates determine each times' level of utility U(.), and the substitute cost is a charge on utility. The social objective is to maximize the sum over time of utility, discounted exponentially at rate r. Uncertainty is accounted for by maximizing the expected value of this objective.

Letting E denote the expectation operator, this is formulated as an optimal control problem:

$$\max_{a(t)} \int_0^T e^{-rt} U(a(t))\, dt \; + \; \underset{h}{E}\, G(Q_1,h)$$

subject to:

$$\begin{aligned} \dot{Q}(t) &= -a(t), \quad 0 \le t < T \\ Q(0) &= Q_0 \\ Q_1 &= Q(T) \\ Q(t), a(t) &\ge 0, \quad 0 \le t < T \end{aligned}$$

where

$$G(Q_1,h) = \max_{\substack{b(t)\\ m(t)}} \int_T^\infty e^{-rt} \left[U(b(t)+m(t)) - h.m(t)\right] dt$$

subject to:

$$\begin{aligned} \dot{Q}(t) &= -b(t), \quad T \le t \\ Q(T) &= Q_1 \\ Q(t), b(t), m(t) &\ge 0, \quad t \ge T. \end{aligned}$$

Let U(.) be continuous and twice differentiable with

$$U' > 0, \qquad U'' < 0, \qquad \lim_{a \to 0} U'(a) = \infty.$$

The cost distribution is bounded allowing derivation of bounds to substitute use rates, which eventually will equalize the substitute cost and marginal utility; by assumption some substitute use is always worthwhile.

The optimum strategy

The optimum precommitment strategy reduces to two deterministic extraction paths: one before T, the other calculated at T when the cost becomes known to society. Depending on the size of the initial stock and on the uncertainty about the substitute cost, a jump in resource use may occur at T. Before T, the optimum balances the current marginal value of resource use with the expected marginal value of resource use after T, satisfying:

$$e^{-rt}U'(a(t)) = \underset{h}{E}\left[\frac{\partial G}{\partial Q1}\right], \quad \text{(a constant)}, \quad \text{all } 0 \leq t \leq T.$$

Possible optimal strategies are illustrated in Figure 3.1. The range K-L of possible optimal substitute use rates is derived from the range of possible costs, and it is assumed that the cost h revealed at T makes use rate H optimal.

AA'HJ is optimal for a small initial stock, exhausted at T. Along AA' the resource use falls, but the discounted marginal utility of resource use is constant.

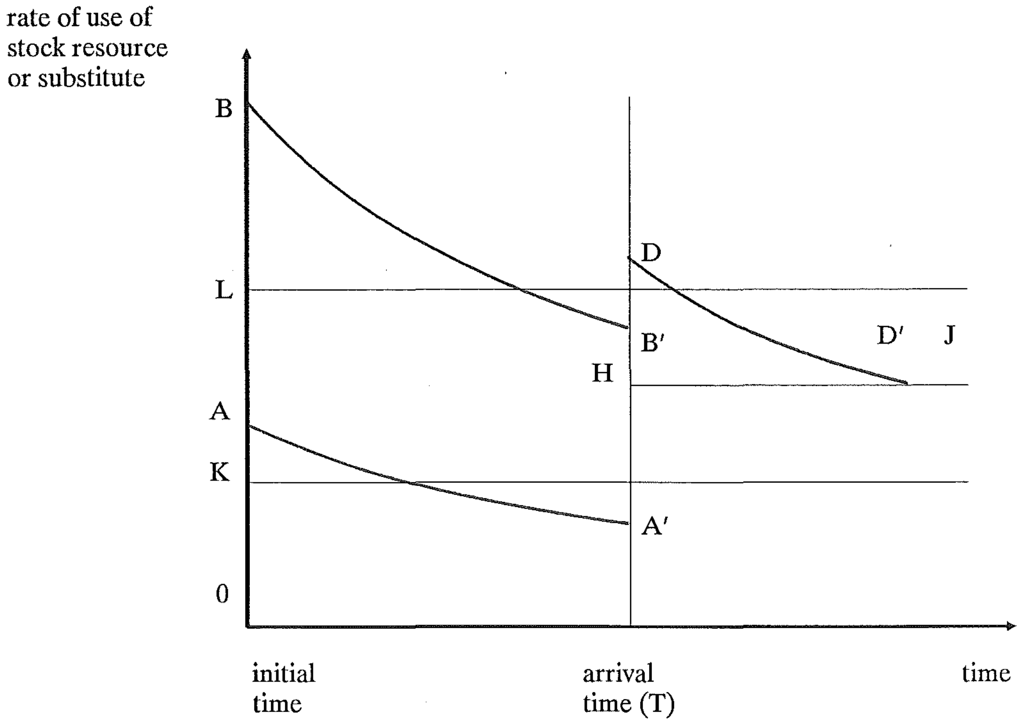

Figure 3.1: Optimal depletion with uncertain substitute cost

BB′DD′J is optimal for a larger stock, exhausted at D′. Along BB′ the discounted marginal utility of resource use is constant, and resource use falls. High initial use rates do not exhaust the resource stock before T: it is better if some stock is kept for use after T. The optimal resource use after T depends on the substitute cost. Any remaining stock is used to keep discounted marginal utility constant for as long as possible. The optimal resource use drops to zero from D′ when the resource stock is exhausted. Substitute use then begins, and continues at rate H.

Reactions to risk-aversion and uncertainty about costs

Because G is convex in h for all Q_1, a mean-preserving increase in cost uncertainty increases optimal resource use before T for a program risk-neutral society. Program risk-aversion reduces extraction at all times before T.

The optimal strategic use of stocks may be faster or slower than if the mean cost outcome is assumed.

3.3.1.2 Cost uncertainty resolved at a chosen time

Alternatively, T is a variable. The substitute cost is discovered when and if the initial stock is exhausted. The value of the remaining program is, like G, positive, decreasing and convex over costs. Assuming risk-neutrality, uncertainty about costs at the exhaustion date translates into a higher expected reward than if the mean cost occurred with certainty. Therefore, optimal extraction is larger and ends sooner under cost uncertainty (Oren and Powell, 1985).

3.3.1.3 Cost uncertainty with capital accumulation

When capital and the resource are inputs to production of a capital-consumption good, which may also be required in producing the substitute, uncertainty about costs is captured with a probability distribution over the input requirements for substitute production.

Under risk-neutrality the optimum initial resource use and capital formation are greater under uncertainty than under certainty with the same mean. However under extreme risk-aversion (which avoids worst possible outcomes) the optimum initial resource use and capital formation are lower than under risk-neutrality (Hanson, 1977).

3.3.1.4 Uncertainty about substitute flows

Uncertainty about substitutes can focus on physical availability instead of cost. In one case, the level m(t) equals a constant k, which is initially uncertain and becomes known at T, rather than being chosen. If the substitute is costless, the deterministic optimal value function $G(Q_1,h)$ is replaced with $F(Q_1,k)$ in the initial objective function, where:

$$F(Q_1, k) = \max_{b(t)} \int_T^{\infty} e^{-rt} U(b(t)+k) \ dt$$

$$\text{subject to:} \qquad \begin{aligned} \dot{Q}(t) &= -b(t), \quad T \le t \\ Q(T) &= Q_1 \\ Q(t), b(t) &\ge 0, \quad T \le t \end{aligned}$$

At optimality, before T, resource use a(t) satisfies:

$$e^{-rt}U'(a(t)) = \underset{k}{E}\left[\frac{\partial F}{\partial Q_1}\right], \quad 0 \le t \le T.$$

Under risk neutrality, a mean-preserving increase in uncertainty about k reduces optimal non-renewable resource use at all times before T (Heal, 1979). This effect is opposite that for cost uncertainty.

3.3.1.5 Conclusions

When uncertainty is focused on substitute costs the value of the remaining program at T is a **positive, decreasing, convex** function of the substitute cost. By Jensen's inequality, a mean-preserving increase in uncertainty about costs increases the expected value of the remaining program at T, so it appears less worthwhile to keep stocks for use after T. Therefore use rates before T increase as substitute cost uncertainty increases.

However, when uncertainty is focused on physical availability, the value of the remaining program at T is a **positive, increasing, concave** function of the level of substitute availability. By Jensen's inequality a mean-preserving increase in uncertainty about availability makes it appear more worthwhile to keep stocks for use after T. Use of the resource before T decreases.

There is no explicit contradiction between these cases, but either cost or availability level could be used to approximate uncertainty about substitutes. The findings are extremely sensitive to the dimension chosen in this approximation. Not even the sign of the impact of uncertainty can be inferred with generality.

Under program risk-neutrality the optimal precommitment strategy is intertemporally consistent, because the objective form here is stationary, and the linear risk transformation does not alter the order over possible programs. The precommitment strategy under program risk-aversion is intertemporally inconsistent, assuming that later generations before T apply the same risk transformation: the continuations possible from $t < T$ are ordered differently at t than at $t' < t$.

3.3.2 The substitute arrival date

Optimal non-renewable resource use, when at some exogenously uncertain time T a technological change provides a substitute for the resource, has been investigated using several different sets of assumptions on technology.

3.3.2.1 Date uncertainty with capital accumulation

In one set of technological assumptions a positive non-renewable resource input a(t), and capital stock K(t), are necessary for production of a consumption-capital good at a positive rate F(K(t),a(t)). The consumption rate c(t), and additions to the capital stock, use all production. At T, a new technology provides a constant flow m of a perfect substitute for the resource.

The previously specified discounted utilitarian objective is applied here to c(t). Given the technology, and the unbounded marginal utility of consumption, any strategy which has a chance of exhausting the non-renewable resource before the substitute arrives cannot be optimal. To avoid this leading to the non-existence of an optimal strategy it is assumed that the substitute is certain to arrive eventually (Dasgupta and Heal, 1974).

Letting the probability density function on arrival times T be n(T), the resource use problem is in formal terms:

$$\max_{\substack{a(t)\\c(t)}} \int_0^\infty n(T) \left(\int_0^T e^{-rt} U(c(t))\, dt + e^{-rT} W(K(T), Q(T)) \right) dT$$

subject to:

$$\dot{K}(t) = F(K(t), a(t)) - c(t) \quad \text{all } t \geq 0$$

$$\dot{Q}(t) = -a(t) \quad \text{all } t \geq 0$$

$$Q(0) = Q_0, \quad K(0) = K_0$$

$$K, c, a, Q \geq 0 \quad \text{all } t \geq 0$$

where

$$W(K(T), Q(T)) = \max_{b(t)} \int_T^\infty e^{-r(t-T)} U(c(t))\, dt$$

subject to:

$$\dot{K}(t) = F(K(t), b(t)) - c(t) \quad \text{all } t \geq T$$

$$\dot{Q}(t) = -b(t) + m \quad \text{all } t \geq T$$

$$K(T), \; Q(T) \text{ given}$$

$$K, b, c, Q, \geq 0 \quad \text{all } t \geq T$$

The nature of an optimal strategy

The optimal strategy consists of two time-streams of resource use rates: one is followed until the arrival time T, when the resource use rate jumps to the other.

For a linear homogeneous production function, resource use and consumption at each time before arrival satisfy the equations:

$$\frac{\dot{c}}{c} = \frac{F_K - r + (n/N).((W_K - U'(c))/U'(c))}{-cU''(c)/U'(c)}$$

$$\dot{x} = z.f(x).(1 + (W_K\, n/N)/(U'(c).f'(x)) + (W_Q\, n/N)/(xf''(x)U'(c))$$

where $x = K/a$,

$f(x) = F(K/a, 1)$,

$z = -f'(x)(f(x)-xf'(x))/xf(x)f''D(x)$,
the elasticity of substitution between K and a,

and N(t) is the probability of arrival no sooner than t.

"The nature of the path (these equations) define is far from obvious except in some special cases." (Dasgupta and Heal, 1974, p21). Given specific functional forms and values for all the parameters it would be difficult to compute the time-streams a(t) and c(t) which satisfy these conditions. The complex optimal strategy must be simplified by assumptions before any insights are available.

An 'unlimited energy' assumption

If the existing stocks of capital and resource become worthless at T, the equations become:

$$\frac{\dot{c}}{c} = \frac{f'(x) - (r + n/N)}{-cU''(c)/U'(c)}$$

$$\dot{x} = zf(x) \quad \text{all } t \geq 0$$

Further specifying an iso-elastic utility function, and a Cobb-Douglas production function, gives the optimal time-streams of Figure 3.2.

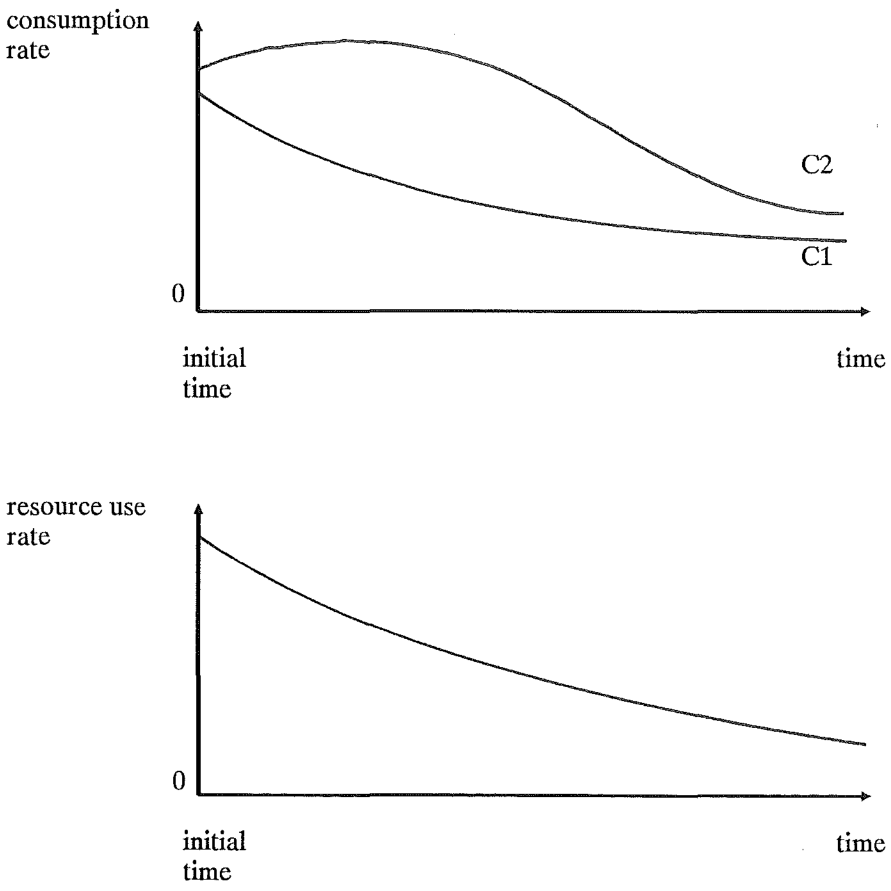

Figure 3.2: Optimal resource use before substitute arrival

Before T the high resource use rates provide high consumption rates (c1 or c2), which may increase for a time (c2) if capital is sufficiently productive. The rapid drawdown of the stock must eventually decrease, because reserves must be conserved against the chance of non-arrival of the substitute. Eventually, if the substitute does not arrive, consumption must fall. When the substitute arrives the time-streams depicted are abandoned and the consumption rates are dictated by the new technology.

Approximating uncertainty by augmented discounting

The optimal strategy above is the same as the optimal strategy when there is no substitute but the discount rate is augmented by n/N, which may well vary over time. Therefore, the conditions which give augmented discounting for uncertainty some theoretical justification are most stringent. If the resource has value after the substitute arrives, augmenting the discount rate for risk is likely to wrongly increase initial resource use and consumption rates.

Approximating uncertainty by the expected situation

Deterministic optimization based on the expected arrival date T_e can be compared with the strategic result. If resource and capital stocks become worthless with arrival, the strategic cumulative resource use must be less than the deterministic cumulative resource use at and after T_e: at T_e the stock is exhausted in the deterministic case but in the strategic case allowance must be made for possible non-arrival of the substitute.

Before T_e things are less clear: "...leaving a resource underground alleviates society from the risk of facing a resource scarcity in the near future, there is an incentive to postpone extraction,... But leaving the resource underground is risky, because the invention will reduce its value... There is then an ambiguity in the implications of uncertainty about the invention date on socially efficient extraction rates. It transpires that whether such uncertainty ought to lead the economy to extract it at a faster or slower rate depends on the size of the initial stock." (Dasgupta and Heal, 1979, p396). This dependency has been demonstrated with no capital accumulation, and where the resource remains valuable after discovery of a substitute (Dasgupta and Stiglitz, 1976, 1981).

The bias inherent in using the deterministic optimum for the expected situation, in place of the strategic optimum, can cause initial resource use to be too fast or too slow.

3.3.2.2 Date uncertainty and Rawlsian justice

The 'justice' of the precommitment strategy, for resource use before substitute arrival, can be examined with versions of the Rawlsian maximin criterion. According to one definition the strategy is 'just' if:

$$J(0) \leq J(t), \quad \text{all } t > 0$$

where $J(t)$ is the expected value of generation t's objective, under the precommitment strategy.

When every generation's objective is to maximize the expected value of total discounted future utilities, and the objective is stationary, the precommitment strategy is 'just' if the initial resource stock is smaller than a critical level which is increasing in the post-discovery inflow level (Riley, 1977).

Illustrative time-streams of expected objective levels, $J(t)$ imposed by the precommitment strategy, are shown in Figure 3.3. After T objectives are maintained at level A by the substitute. Time-stream B corresponds to an initial stock below the critical level, so the strategy is 'just': J(t) increases because the underlying fall in planned resource use (given non-arrival of the substitute) is more than offset by the decreasing likelihood of non-arrival of the substitute.

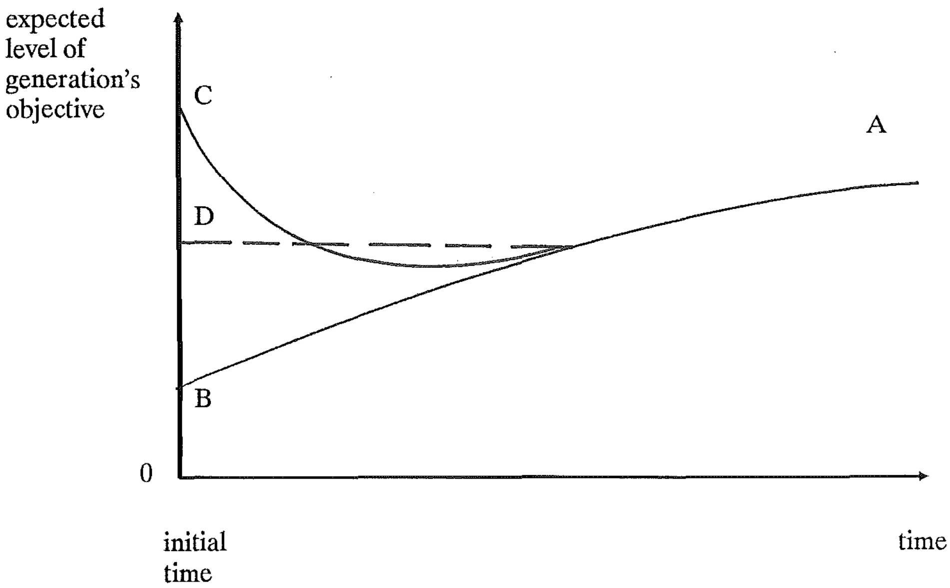

Figure 3.3: Justice and expected objective levels

A strategy which violates the 'justice' constraint underlies time-stream C. Here, a high initial resource use (given non-arrival) falls so fast that J(t) falls as well - the decreasing likelihood of non-arrival of the substitute is not enough to offset falling planned resource use.

When the initial generation's strategy is **constrained** to be 'just', a time-stream like D results. The underlying resource use is initially lower and falls more slowly over time, so that J(t) is constant for some time before eventually increasing with the increasing likelihood that the substitute inflow has arrived.

This implies that intergenerational 'justice' might require early generations to slow resource use, even when their objective covers future generations and even though a substitute is certain to arrive eventually.

The 'justice' constraint on expected values is less stringent than one on (say) minimum possible levels.

3.3.2.3 The possibility of non-arrival

The distribution of arrival dates can be explicitly conditioned on the 'existence' of the substitute technology. The initial probability that the substitute exists decreases as the substitute fails to arrive, until the substitute does arrive if ever (Bhattacharya, 1984).

For the production framework, the optimal initial use of the resource (given non-arrival) increases with the initial probability of existence. Also, the optimal initial use decreases as the instantaneous utility function becomes more risk averse, across the class of instantaneous utility functions.

3.3.2.4 Conclusions

The theoretical investigations demonstrate that the optimal precommitment strategies are 'not obvious' when the arrival date of substitute technology is uncertain and beyond influence. Optimal strategies depend in a complex way on the societal objective, the conception of 'justice', the production possibilities both before and after the discovery, and the way the uncertainty about the arrival date is resolved over time.

In the optimal precommitment strategy, the impact on the initial resource use of increasing uncertainty about the arrival date is unclear, but increasing the initial probability that the substitute will never be available decreases the optimal initial resource use. Basing social choice on a deterministic approximation is unlikely to be justifiable on theoretical grounds. Augmenting the discount rate to allow for possible substitute arrivals is likely to produce a deterministic resource use optimum which is initially larger than the strategic optimum.

As for uncertainty about the nature of substitutes, the investigations here maintain intertemporal consistency of the precommitment strategy by using only stationary utilitarian objectives.

3.3.3 Research and development

Real world substitutes are created by research and development (R&D). These activities use resources with an opportunity cost. The uncertainty in R&D can be approximated by assuming that a stock of 'knowledge' about the substitute is accumulated, and at an uncertain level of this knowledge the substitute becomes feasible. Various input assumptions can be made.

3.3.3.1 A stock input to R&D

If some capital $K_D(t)$ is devoted to research, which increases the knowledge stock $x(t)$ by $D(K_D(t))$, the formulation is:

$$\max_{\substack{a(t)\\ c(t)\\ K_D(t)}} \int_0^\infty n(X) \left(\int_0^{T_X} e^{-rt} U(c(t))\, dt + e^{-rT_X} W(K(T_X), Q(T_X)) \right) dX$$

subject to:

$$\dot{K}(t) = F(K-K_D(t), a(t)) - c(t) \quad \text{all } t \geq 0$$

$$\dot{Q}(t) = -a(t) \quad \text{all } t \geq 0$$

$$\dot{x}(t) = D(K_D(t)) \quad \text{all } t \geq 0$$

T_X is defined by $x(T_X) = X$

$Q(0) = Q_0$, W given

$Q, a, K, K_D, c, x \geq 0 \quad \text{all } t \geq 0$

where W = the value of the program after arrival
and $n(X)$ = the probability density of arrival at knowledge level X.

Letting $N(X)$ be the probability of arrival at a knowledge level no smaller than X, the necessary conditions for optimality can be expressed as:

$$\frac{d}{dt} \log F_a = F_K + W_K(n/N)\, D/U',$$

$$c(t) = c(0) \left[e^{-rt}\, NF_a / F_{a|t=0} \right]^{(-U'/cU'')}$$

$$F_{KD} \frac{d}{dt} \left[\log F_a\, D'/F_{KD} \right] = D'.(n/N) \left[U\text{-}r\ W + \dot{W} \right] /U' .$$

This optimal control formulation is difficult to solve analytically because it has three state and three control variables. Results can be found if the production function is Cobb-Douglas, the arrival level distribution function is negative exponential, and the knowledge production function is linear. Under these assumptions, before discovery, the optimal precommitment resource use rate falls over time, as do consumption and the capital devoted to R&D. Knowledge grows until a cutoff date after which no R&D is performed and there is no further chance of discovery. The R&D cutoff date advances as the discount rate rises. If the initial knowledge level is low enough, it may be optimal to never perform R&D (Davison, 1978).

3.3.3.2 A flow input to R&D

An alternative formulation is that the R&D input is a flow d(t) of produced goods rather than a capital stock. Decreasing returns to the compression of R&D effort are captured with a concave knowledge production function D(d(t)) (Kamien and Schwartz, 1978; Dasgupta *et al.*, 1977).

The revised equations of motion are:

$$\dot{K}(t) = F(K(t),a(t)) - c(t) - d(t) \quad \text{all } t \geq 0$$

and

$$\dot{x}(t) = D(d(t)) \quad \text{all } t \geq 0$$

The optimal precommitment strategy is very different: R&D need not begin immediately, R&D will always eventually begin, consumption and R&D effort may initially increase before tailing off as discovery fails to occur, some R&D will always be continued. The optimal R&D effort falls faster when the discount rate is higher, and falls more slowly when the assumed instantaneous utility function is more risk averse.

Simulation experiments indicate that the optimal R&D start date is extremely sensitive to the productivity of the R&D effort: an immediate R&D start jumps to an arbitrarily late R&D start with a small decrease in productivity. The productivity level at which this jump occurs is lower for lower discount rates and lower when non-renewable stocks are plentiful (Pand, 1975 reported in Dasgupta *et al.*, 1977). The implication is that societies with low discount rates will optimally do early research at low productivities whereas societies with high discount rates would optimally wait. Also, societies with plentiful stocks will optimally do early research at low productivities whereas societies with few stocks would optimally wait.

3.3.3.3 Comparison

The eventual termination of R&D of the stock input formulation is not found for flows, because the latter does not use a linear knowledge production relation which places an upper limit on productivity at low effort levels.

The optimal precommitment is sensitive to the functional form and parameters used: "Clearly the most appropriate formulation can only be determined by empirical research, and at the moment there seems to be little work of relevance to this question. Indeed, one of the striking features of this area is the lack of empirical information about the appropriate functional forms and distributions." (Dasgupta *et al.*, 1977, p503).

The uncertainty about R&D investigated in the theory is of the 'risk' type only. It is clear that beyond this there is a "fundamental conceptual problem... Many of the most important products of research activities have been concepts and techniques that were completely unknown only a quarter of a century prior to their discovery...modelling the rational allocation of resources to the discovery of ideas and techniques that we cannot even conceptualize at present, and of whose potential existence we are completely unaware, is obviously a far more challenging problem. Indeed, it might well prove impossible - though such research, and its results, are obviously an important element in social progress." (*ibid*, p504).

As above, intertemporal consistency of the precommitment strategy is maintained by the stationary utilitarian objectives used.

3.4 Prescriptive investigations of resource uncertainty

The current desirability of use of a non-renewable resource depends on its future availability, which is uncertain. Availability depends on how much more will be found, when it will be found, and what opportunity costs will be incurred if and when the find is extracted.

Availability therefore depends on exploration efforts, and efforts in development of extraction technology. These are likely to vary with the perceived rewards, which are likely to increase as resource use rates and stocks fall. In reality, resource use rates both affect and are affected by estimated future resource availability.

Most of the prescriptive literature consists of investigations of the impact of uncertainty on resource use. The reverse impact is covered as well in a few investigations incorporating exploration. The uncertain availability is approximated by an uncertain discovery size, or extraction (opportunity) cost, or discovery date, as Table 3.2 summarizes.

Table 3.2: Investigations of uncertainty about the resource base

Uncertain item	Uncertain attribute	Type of uncertainty
future new reserves	size of stocks	exogenous
	extraction cost	exogenous
	discovery date	exogenous
	stock size and discovery date	size exogenous, date influenced
current reserves	stock size, and so exhaustion date	size exogenous, exhaustion date influenced

3.4.1 Two deposit approaches

The simplest way to investigate resource base uncertainty is to assume that a non-renewable resource stock Q_0 is available and an additional amount Q_1 of the same resource will later become available (Heal, 1979). Either the additional amount or discovery time T is uncertain; optimal precommitment strategies are sought: the objective is to maximize the expected value of total discounted societal benefits from consuming the resource. This stationary utilitarian form ensures the optimal precommitment is intertemporally consistent for all the two deposit approaches.

Uncertain second deposit size

When the size of the second deposit is uncertain the optimal control formulation is:

$$\max_{a(t)} \int_0^T e^{-rt} U(a(t))\, dt \quad + \quad \underset{Q_1}{E}\, G(Q_1, Q_T)$$

$$\text{subject to:} \qquad \dot{Q}(t) = -a(t) \qquad \text{all } 0 \le t < T$$

$$Q(0) = Q_0, \quad Q_T = Q(T)$$

$$Q, a \ge 0 \qquad \text{all } 0 \le t < T$$

where

$$G(Q_1, Q_T) = \max_{b(t)} \int_T^\infty e^{-rt} U(b(t))\, dt$$

$$\text{subject to:} \qquad Q(T) = Q_T + Q_1$$

$$\dot{Q}(t) = -b(t) \qquad \text{all } t \ge T$$

$$b, Q \ge 0 \qquad \text{all } t \ge T$$

At optimality, if possible given the parameters (Q_0, T, and the distribution on Q_1):

$$e^{-rt} U'(a(t)) = \underset{Q1}{E} \left[\frac{\partial G}{\partial Q_T} \right]$$

$$\text{so} \quad \frac{\dot{a}(t)}{a(t)} = \frac{-rU'(a(t))}{a(t)U''(a(t))} \qquad \text{for } 0 \leq t \leq T,$$

and this is a constant proportional rate of decline if U exhibits constant marginal elasticity of resource use.

After T, a deterministic problem is solved for the optimal use of the discovery plus whatever remains of the initial reserve. These use rates maintain constant discounted marginal utility and in the limit exhaust all resources.

Before T, initial use rates also provide constant discounted marginal utility. The rates are (if possible) set to equate the discounted marginal utility of initial use with the expected discounted marginal utility after discovery. When equality is achieved optimal resource use jumps up or down at T depending on the size of Q_1. The jumps in use rate from A′ to B, C, or D illustrate this adjustment in Figure 3.4.

If Q_0 is too small to equate the initial discounted marginal utility and the expected gain, then Q_0 is exhausted at T, and resource use jumps upwards then as at E′. When marginal utility is unbounded it is not optimal to take a chance on running out at T, although the reserves held as 'insurance' at that date may be very small.

If it is optimal to exhaust Q_0 at T then a mean preserving increase in uncertainty about Q_1 may have no effect on the optimal strategy, since Q_0

may remain 'too small'. If the uncertainty increases so that an empty discovery becomes possible, a slow down in initial resource use is optimal.

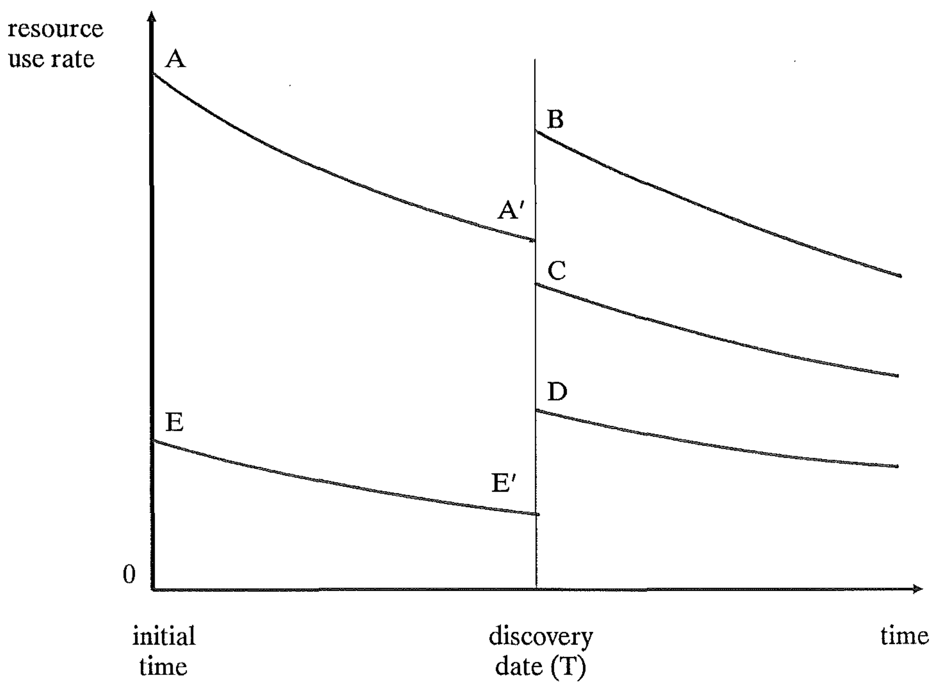

Figure 3.4: Optimal resource use: uncertain second deposit size

If it is not optimal to exhaust Q_0 at T then a mean-preserving increase in uncertainty about Q_1 decreases optimal initial use levels, if the expected marginal value of extra units for later use "...is a convex function (the most likely case as it is bounded below)." (Heal, 1979, p132).

Second deposit revealed at exhaustion of the first

A different formulation assumes that T is a variable to be chosen. The size Q_1 and extraction 'cost' c of the second deposit are uncertain and are revealed when the first deposit is exhausted at T (Hoel, 1978b). Costs are a charge on utility.

The overall objective is now:

$$\underset{\substack{a(t)\\T}}{\text{Max}} \int_0^T e^{-rt}\, U(a(t))\, dt \quad + \quad \underset{(Q_1,c)}{E}\; G(Q_1,c)$$

where

$$G(Q_1,c) = \quad \underset{b(t)}{\text{Max}} \int_T^{\infty} e^{-rt} \left(U(b(t)) - c.b(t) \right) dt$$

subject to: $\quad \dot{Q}(t) = -b(t) \quad \text{all } t \geq T$

$$Q(T) = Q_1$$

$$Q, b \geq 0 \qquad \text{all } t \geq T.$$

Two deterministic paths form the solution: one for the known reserve, and one for whatever additional reserve is discovered. It may be optimal not to exhaust the known reserve if the discovery may be empty.

It is most plausible that G is **concave** in Q_1, so that a mean-preserving increase in uncertainty about Q_1 reduces the expected value of $G(Q_1,c)$, and consequently the optimal initial extraction path is lower and T is larger. Similarly, if G is **convex** in c, a mean-preserving increase in uncertainty about c increases the expected value of $G(Q_1,c)$, and consequently the optimal initial extraction path is higher and T is smaller. This is illustrated in Figure 3.5.

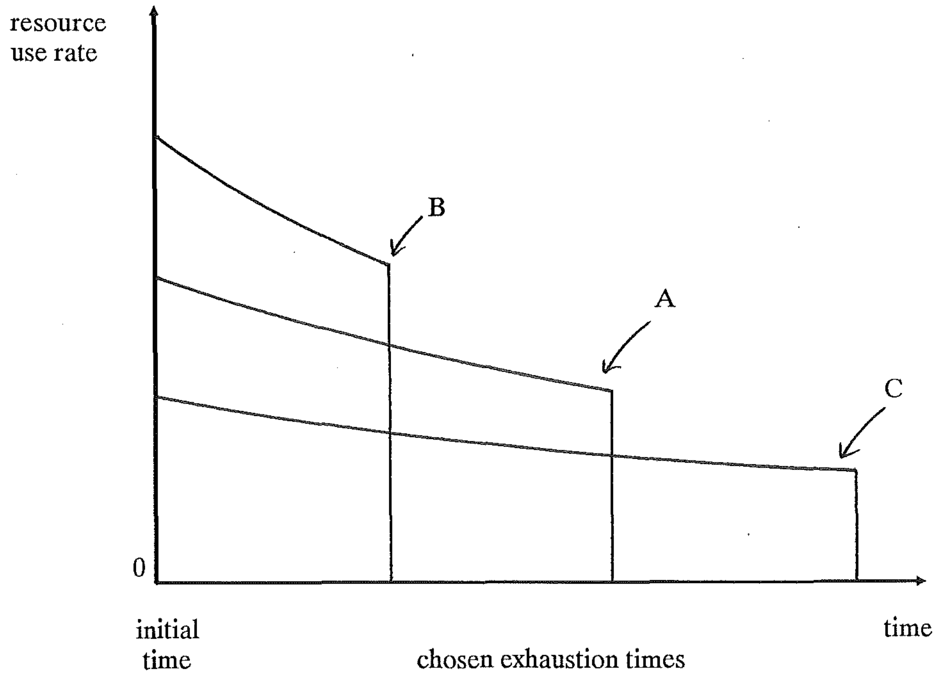

Figure 3.5: Optimal resource use: uncertain second deposit size or cost

The optimal path tends from A towards B as the uncertainty about the 'cost' of the discovery increases, and tends from A towards C as the uncertainty about the discovery size increases. The conclusion parallels that for flow discoveries:

"...uncertainty with respect to the size of the resource stock may affect the extraction path in a way which differs from uncertainty with respect to the future extraction cost. [...] Economic models with a fixed and finite supply of a homogeneous natural resource are sometimes regarded as a simplified representation of the more realistic case where the natural resource is becoming more costly to extract as it is depleted, but it is never completely exhausted. [...] when uncertainty is treated the two descriptions of natural resource scarcity have different implications."(*ibid*, p645).

Uncertain second deposit discovery date

The framework can be used to investigate the discovery of a known amount Q_1 of resource at an unknown time T:

$$\underset{a(t)}{\text{Max}} \int_0^\infty n(T) \left(\int_0^T e^{-rt} U(a(t))\, dt + e^{-rT} W(Q_T) \right) dT$$

subject to:

$$\dot{Q}(t) = -a(t), \quad \text{all } t \geq 0$$
$$Q(0) = Q_0, \quad Q_T = Q(T),$$
$$Q, a \geq 0, \quad \text{all } t \geq 0$$

where

$$W(Q_T) = \text{Max} \int_T^\infty e^{-r(t-T)} U(b(t))\, dt$$

subject to:

$$\dot{Q}(t) = -b(t) \quad \text{all } t \geq T$$

$$Q(T) = Q_T + Q_1$$

$$Q, b \geq 0 \quad \text{all } t \geq T$$

After T, remaining reserves Q_T plus Q_1 are used so that they are exhausted in the limit and the discounted marginal utility of resource use is constant.

Before T the optimal precommitment use rates at all times satisfy:

$$\frac{\dot{a}}{a} = \frac{r}{m} + \frac{n/N\,(1 - W'/U')}{m}$$

where $m = \dfrac{-aU''(a)}{U'(a)}$, the marginal elasticity of utility of resource use.

Given that n/N is the chance of a discovery in the next instant when no discovery has yet been made, at times with no chance of discovery n/N is zero, and the discounted marginal utility is constant.

Otherwise, uncertainty augments the effective discount rate by an amount which depends on U′ and W′. This augmentation will only be constant where the discovery time possibilities follow a negative exponential distribution, so that n/N is constant, **and** Q_1 is so large relative to the initial stock that discovery renders any remaining initial stock valueless ($W'/U' = 0$).

Optimal consumption monotonically declines whatever the chance of discovery. Also, the initial reserve can only be exhausted at a finite time along the optimal time-stream if the discovery is certain to have occurred by this time.

Heal (1979) claims that a mean-preserving increase in uncertainty about the discovery date causes the optimal initial resource use to be smaller. This result does not hold for at least one mean-preserving increase: from certainty to a negative exponential distribution.

3.4.2 Gradually resolved resource uncertainty

3.4.2.1 Cake-eating under uncertainty

One widely investigated model assumes that the current uncertainty about the size of the resource stock is resolved at exhaustion, when it is discovered that the unit just used was the last unit which will ever be available (Cropper, 1976; Kemp, 1976; Loury, 1978; Gilbert, 1979; Heal, 1979; Kasanen, 1982). An alternative interpretation is that the inevitable waste disposal associated with production and consumption is using up a finite ‘sink’ of the environment’s assimilative capability. This sink will eventually be filled to capacity and economic activity will have to cease, but the capacity itself is uncertain (Cropper, 1976).

Stock sizes and exhaustion dates

The central optimal control formulation is:

$$\max_{a(t)} \int_0^\infty e^{-rt}\, U(a(t))\, P(Q(t))\, dt$$

subject to:

$$\dot{Q}(t) = a(t) \quad \text{all } t \geq 0$$
$$Q(0) = 0$$
$$a(t) \geq 0 \quad \text{all } t \geq 0$$

where Q is now cumulative consumption and P(Q(t)) is the initial (prior) probability that the resource stock is at least as big as Q(t).

The optimum precommitment strategy a(t) is followed until the initial stock is exhausted, at which time resource use drops to zero and remains there. Given the probability distribution over possible initial stock sizes, every strategy produces a probability distribution over dates of exhaustion.

The usual stationary utilitarian objective is assumed, but it is critical that instantaneous utility is bounded below as consumption falls to zero. Otherwise, no chance of the resource running out can be taken, and the solution involves spreading a known minimum amount of the resource (if there is one) over an infinite time, which is a deterministic problem.

The optimal strategy

The optimal strategy ensures that at all t the discounted marginal utility of resource use equals the expected value of the discounted average utility of resource use at T, the end of the program (Loury, 1978). The expectation is taken over the probability distribution on T conditioned on reaching t. Formally:

$$e^{-rt}\, U'(a(t)) = \underset{T}{E}\, [e^{-rT}\, U(a(T))/a(T) : T \geq t]$$

At any t a marginal increase in a(t) has the payoff $e^{-rt}\, U'(a(t))$. A marginal increase in a(t) advances T, given that the optimal path is followed subsequently. Suppose exhaustion would have occurred at time $s \geq t$. Then the reduction in the duration of the program due to a unit increase in a(t) is 1/a(s). The associated societal loss is $e^{-rs}\, U(a(s))/a(s)$. At t the time s is unknown, so the expected discounted utility loss over possible values of s is balanced with the payoff. That is, along the optimum time-stream, the marginal gain to increased consumption at each time is just balanced by the expected marginal cost due to lost consumption at the end of the life of the resource.

Equivalently, suppressing the time arguments of a and Q, it is necessary that at all times:

$$\frac{\dot{a}}{a} = \frac{\left([(U(a)/a - U'(a))/U'(a)].a.P(Q) \right) - r}{m(a)}$$

where P is the conditional probability density on the reserve running out at Q, given that it is at least as big as Q.

For concave utility functions that are bounded below the term in square brackets is always positive, as must be P. The right-hand-side can therefore be positive and/or negative at times along the optimal path; uncertainty can reduce the effective discount rate. Optimal resource use may rise - especially when P, the probability of running out on the next unit, is large - or fall. This is in contrast with the deterministic cake-eating result that optimal resource use falls monotonically over time.

Increasing the discount rate to allow for risk, while dealing with the best deterministic approximation to the resource size, cannot reproduce an optimum involving increasing consumption, and so does not have general theoretical backing.

Solution sensitivity

An increase in r makes optimal resource use rates fall faster when they are falling and rise more slowly when they are rising. Cumulative use along the optimal time-stream always tends in the limit to the largest possible resource size (Loury, 1978, p625). Increasing the discount rate therefore shifts the strategy for optimal use from later times to earlier times.

When an uncertain resource base is compared with a resource stock equal to its mean, the optimal precommitment strategy ensures that the expected stock on hand in the uncertain case is always as big as for the certain case. The optimal initial use rate will be less for the uncertain case, but in general use rates for the uncertain case can be larger or smaller than for certainty. The eventual drop to zero resource use can be avoided with known reserves, so this uncertainty may be said to unavoidably impose losses on future generations (Cropper, 1976; Loury, 1978; Gilbert, 1979).

Results are unclear when two uncertain resource stocks are compared. For mean-preserving increases in uncertainty about stock sizes the initial optimal use for the more uncertain case can exceed that of the less uncertain case. Heal (1979, p140) wrongly claims that this is a general result. Loury demonstrates that it can occur (1978, p629), but refers to it as a "perverse effect" occurring under unusual circumstances, presumably because it goes against the usual intuition that more uncertainty should lead to more cautious use.

3.4.2.2 Alternative objectives

It is well accepted that: "Custom condones the choice of the sum of discounted expected utilities as maximand. However, custom is based on considerations of tractability rather than plausibility ..." (Kemp 1976 p301). Two other objectives have been examined. The first allows that utility may depend on cumulative past consumption as well as current consumption: $U \equiv U(Q,a)$. The optimal strategy for resource use then may also produce a time-stream of resource use rates which rises and falls over time (*ibid*).

The second objective is a version of the Rawlsian maxi-min criterion:

$$\underset{a(t)}{\text{Maximize}} \; \underset{t}{\text{Min}} \; \underset{s}{E}[U(a(t))] = \underset{a(t)}{\text{MAX}} \; \underset{t}{\text{MIN}} \; [U(a(t)).P(Q(t))]$$

where

E	is the expectation operator,
s	is the possibility space: 'still going', or 'run out',
a	is planned consumption at t,
Q	is planned cumulative consumption before t,
P	is the probability that the resource runs out at or after reaching cumulative consumption Q(t).

Kemp states: "along the optimal path,

$$U(a(t)).P(Q(t)) = U(\dot{Q}(t)).P(Q(t)) = \text{a constant}$$

implying that consumption is either zero or is growing. [if the stock size possibilities are bounded above]... then optimal consumption is zero. Only when there is a positive probability of the cake being of any size, however large, is it optimal to consume any of it - and in that case consumption must grow." (*ibid*, p302).

This is not correct, since the objective attains the same value for all paths which reach zero resource use (with probability one). If no path can avoid this then all use paths are alternative optima along with the zero use path. The maximin maximand, as formulated by Kemp, has little discriminatory power for this problem.

3.4.2.3 Multiple uncertain deposits

Few results have been derived for situations with multiple uncertain deposits. The optimal control formulation and solution methods apply, but the complexity of the situation means clear analytical results are unavailable. Optimal precommitment resource uses can increase and decrease over time in the general multiple uncertain deposit case (Kemp, 1977). Optimal use of a number of uncertain deposits which must be taken in fixed order is a cake-eating problem under uncertainty. The same utility bounds are required, and the same generally conservative outcome relative to certainty is found (Gilbert, 1979).

The impact of uncertainty on the optimal precommitment sequence for use of multiple deposits cannot be easily characterized except in some special cases (Robson, 1979; Hartwick, 1983). If there are two costless deposits, one of known size and one uncertain, the uncertain deposit should be used to exhaustion before the known deposit is used at all, irrespective of the utility form and the probability distribution covering possibilities. This follows from the value of being able to make informed plans when exhaustion of the uncertain deposit occurs. When both deposits are of uncertain size it is the exception for it to be optimal to completely exhaust one deposit before the other, and no simple characterization of the depletion policy is available.

Throughout the cake-eating investigations, intertemporal consistency of the precommitment strategy is ensured by the stationary utilitarian objective in all cases but one, and that maximin objective has no discriminatory power for the cake-eating situation.

3.4.3 Exploration

In reality, resource base uncertainties are resolved gradually and deliberately by exploration, which cannot be discussed in pure cake-eating models.

3.4.3.1 Extraction in advance of need

If extracted resource can be stored then a type of 'exploration' can be introduced to the cake-eating problem: uncertainty is resolved when no further extraction is possible, just after cumulative extraction reaches one of a finite number of possible sizes. Exploration involves extracting resources in advance of need, to determine whether exhaustion is about to occur (Gilbert, 1979).

The exploration 'costs', a charge on utility, consist of the interest cost of extracting resources before they are used, plus a storage cost. Exploration benefits result from having some stock in storage when extraction becomes impossible, so that utility levels can be maintained to some extent after the resource is exhausted.

The optimal precommitment utilitarian strategy always involves 'exploration' if the storage cost is zero, or if the marginal utility of resource use is unbounded. Other storage cost and marginal utility combinations can also make exploration worthwhile. An increase in the cost of extraction or storage delays the time at which exploration will start.

3.4.3.2 Exploration with unlimited territory

Exploration can be treated as an activity which transforms unexplored territory into reserves of a non-renewable resource.

Assumptions

Two exploration uncertainties have been investigated: an uncertain time until a discovery is made, and discoveries of uncertain size. Probability distributions over timing and size, conditional on the exploratory effort, are exogenously given. In one investigation the distributions are independent of unexplored territories, so the resource base is implicitly inexhaustible. Exploration costs are a charge on utility.

The formulation assumes that discoveries form a Markov process, and shows that the reserves level Q(t) does also, for exploration and use policies which are stationary functions of the reserve level: x(Q(t)) and a(Q(t)). Markovian decision theory then allows the optimal policies to be identified.

Letting h be the exploration cost function, Y be the set of policies (x(Q),a(Q)), and E_y be the expectation given the Markov process induced by $y \in Y$, the objective is:

$$\max_{y \in Y} V_y(Q_0) = E_y \left[\int_0^{\infty} e^{-rt} \left[U(a(Q(t)) - h(x(Q(t)) \right] \, dt \right]$$

subject to: $Q(0) = Q_0,$

$Q, a, x \geq 0 \quad \text{all } t \geq 0$

Also,

$$dQ = -a\,dt + B(x(Q), \cdot), \quad \text{all } t \geq 0.$$

where B is the (stochastic) discovery rate for finds of various sizes when exploring at level x. Dynamic programming arguments establish that the optimal policy y* is associated with an optimal value function V(Q).

The nature of an optimal strategy

The optimal precommitment strategy here is open-ended: in the 'typical' case exploration will be building up reserves through an occasional discovery, and consumption will be using up reserves through extraction, at all times.

The optimal extraction rate is a non-decreasing stationary function of the reserve level, and the optimal exploration rate is a non-increasing stationary function of the reserve level (Deshmukh and Pliska, 1980, 1983).

The extraction, exploration, and reserve level paths which might result are illustrated in Figure 3.6.

A small discovery is made at A, a larger one at C, and a still larger one at B: the discovery is added to proven reserves, and this extra security allows the consumption rate to be increased and the exploration rate to decrease, with the big discovery at B allowing a temporary cessation of exploration. Between discovery times consumption steadily lowers proven reserves, which leads to a steady increase in the exploration rate, and a steady decrease in the consumption rate.

The outcomes attainable depend greatly on the utility, exploration cost, and probability measure forms. The optimal strategy might involve extraction rising to an upper limit and remaining there, or falling to zero and remaining there (U′(0) may be finite). More specific results on the shape of the optimal policy require more assumptions on the underlying functional forms.

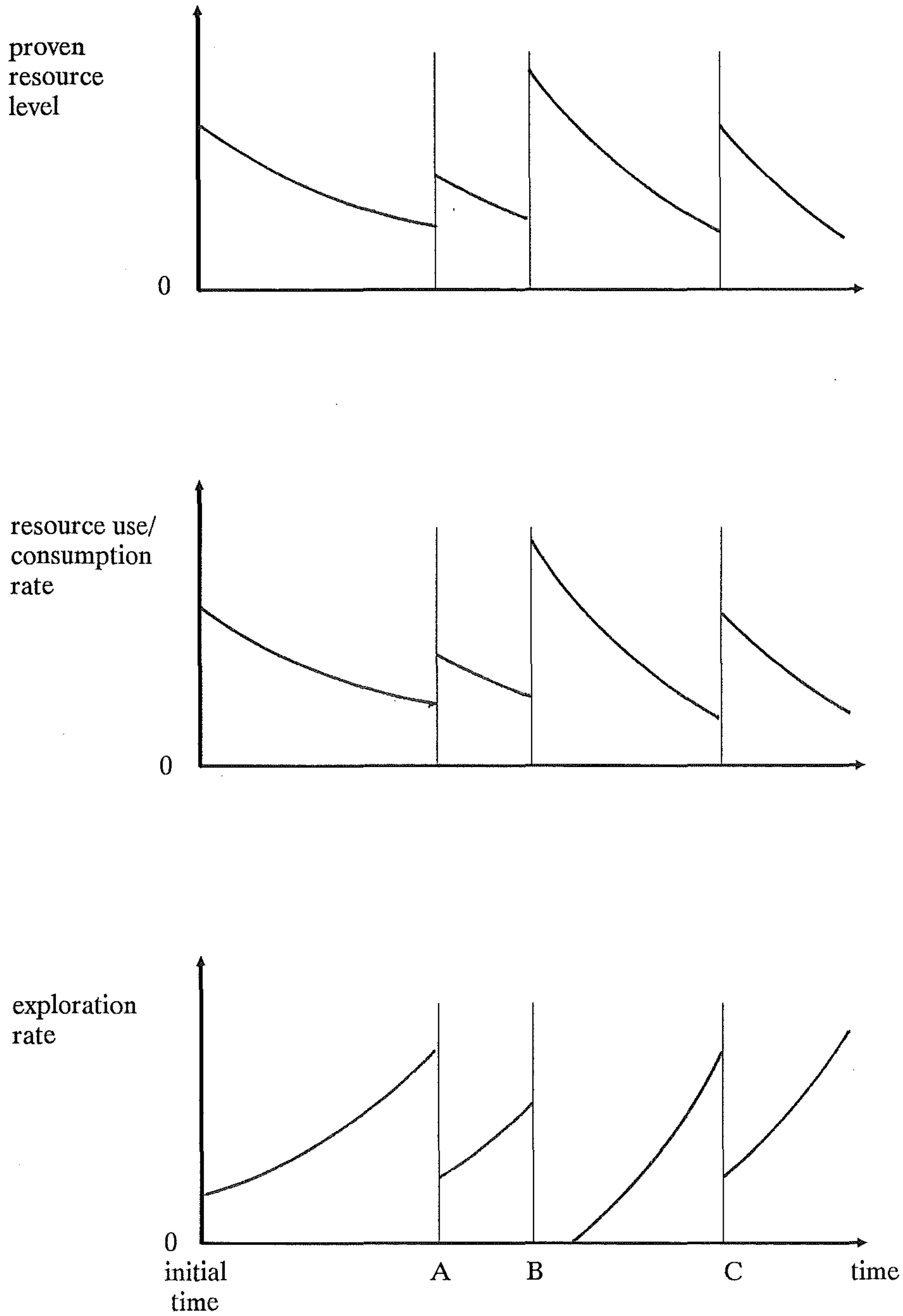

Figure 3.6: Optimal paths with stationary uncertain exploration

Economic interpretation

At optimality $U'(a(Q(t))) = V'(Q)$, i.e. the marginal utility of resource use is equal to the shadow rent on proven reserves, that is the increase in the expected discounted return given another unit of reserves. Also, the marginal exploration cost is equal to the marginal expected increase in the discounted return from using discovered reserves.

With strictly concave utility and strictly convex exploration costs, the expected marginal value of proven reserves, and hence the marginal utility of extraction/consumption, is expected to rise exponentially at the discount rate whenever reserves are positive:

$$\lim_{t \to 0} \frac{E_{y^*}\left(V'(Q(t)) \mid Q(0) = Q_0\right) - V'(Q_0)}{t} = r\,V'(Q_0)$$

This expected increase is "...a sort of local phenomenon, but does not necessarily imply that the [actual expected marginal value of proven reserves] will grow indefinitely (in any probabilistic sense) as time goes on."(*ibid*, p 193). The expected marginal value of proven reserves is renormalized by a downwards jump every time a discovery is made.

3.4.3.3 Exploration with limited territory

Two state variables are required to investigate the exploration of a limited territory: the level of proven reserves and the level of unexplored territory. Learning is limited to establishing how many deposits (each of known size) are present in each piece of territory.

Three other assumptions are critical: no upper bound is placed on the level of exploration effort; exploration costs and the discovery rate are proportional to the exploration effort. Exploration 'costs' are a charge on utility, and a stationary utilitarian objective is adopted (Arrow and Chang, 1982).

The policies y are now stationary functions in two state variables $(a(Q,L), x(Q,L))$; where L is the remaining unexplored territory, and the objective is:

find

$$V(Q_0,L_0) = \max_{y \in Y} E_y \int_0^\infty e^{-rt} (U(a(t)) - h.x(t))\, dt$$

so that by dynamic programming arguments, at optimality (given suitable differentiability):

$$r.V(Q,L) = \max_{y \in Y} [\ U(a)- h.x- V_Q\, (Q_0,L).a- V_L(Q,L_0).x + pxdV\ ]$$

where

$$dV = V(Q+1,L) - V(Q,L)$$

and p is the discovery rate parameter.

Interpreting the optimal exploration strategy

The optimal exploration rate is at each time either zero or infinite. One interpretation of 'infinite' exploration is internally consistent. At any time an 'exploration event' may be undertaken. With each such event one deposit will certainly be found if one or more remains in the remaining unexplored territory. Exploration is undertaken only to the extent required for one deposit to be found, so the reduction in unexplored territory due to an exploration event is a random variable with outcome equal to the amount of territory which must be explored to find one deposit. The remaining unexplored territory drops in a jump at each exploration time.

However, a property of events governed by the Poisson distribution (that the number of events occurring in a very small time interval is either zero or one) is frequently used in this analysis. This property is inconsistent with an 'infinite' exploration rate. The analysis must be "...largely heuristic..." (*ibid*, p2).

The optimal strategy: suggested nature

Given the above rider, the optimal extraction rate is positive whenever proven reserves are positive, decreases over time between exploration events, and jumps up with each increase in reserves. Figure 3.7 illustrates this.

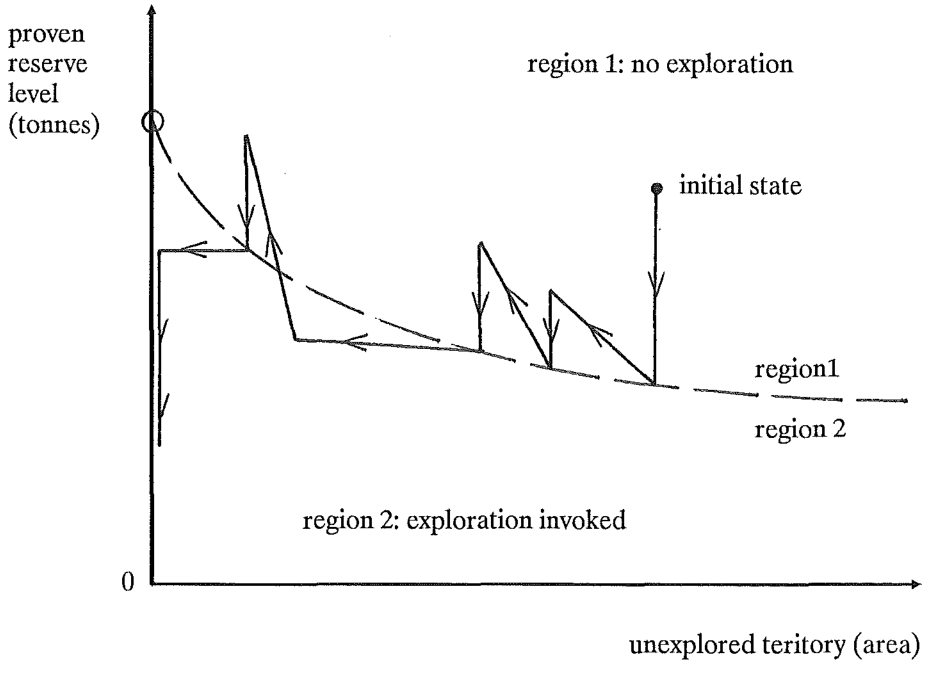

Figure 3.7: State transitions with optimal exploration

In Region 1 of the state space no exploration is undertaken and extraction reduces reserves over time. Eventually the state enters Region 2, exploration is invoked, the reserve level jumps up and in unexplored territory jumps downwards. This jump returns the state to the no-exploration region where the optimal strategy provides a new extraction level.

There is no passage of time in any state transition with a horizontal element in Figure 3.7. The reserve threshold for exploration increases as unexplored territory decreases, so the boundary between the regions slopes downward.

Economic interpretation

The expected marginal value of proven reserves (EMVPR) increases exponentially over time at the discount rate while the state is in Region 1. The EMVPR jumps downwards with each exploration: "The [overall shift in the EMVPR] will show fluctuations with little upward trend when [the amount of unexplored territory] is large; presumably the upward trend is stronger as [the amount of unexplored territory] approaches zero." (Arrow and Chang, 1982, p10).

An extension

The exploration policy might require more than one exploration event to return the state variables to Region 2. Also, the EMVPR and the expected marginal value of unexplored territory are **both** expected to fall at the time of each exploration event. Exploration can be sufficiently unlucky, through having to explore a large amount of territory, to lead to an upwards jump in the EMVPR. The last exploration event, which exhausts the unexplored territory, certainly causes an upwards jump in the EMVPR, because the reserve level is not increased but the hope of finding more reserves is eliminated (Lasserre, 1984).

Throughout the investigation of exploration intertemporal consistency of the precommitment strategies is ensured by use of stationary utilitarian objectives.

3.5 Numerical investigations

Some investigations which seek to inform decision-makers about principles for use in non-renewable resource decisions are on the boundary between theoretical and applied modelling. Most of this work centres on energy supplies. Here numerical solutions are obtained by mathematical programming methods.

3.5.1 Sectoral optimization models

Sectoral models employ the partial equilibrium assumption and optimize resource activities, subject to constraints on resource availability and so as to meet demand for end-use products. Demand levels or curves are exogenously given, and the respective objectives are to minimize total discounted costs or maximize total discounted consumer plus producer surplus. By comparison with the approaches of sections 3.2 these models ignore resource ownership, but consider more technological detail. Resource scarcity enters through constraints, and uncertainty is generally explored with scenarios.

Issues in uranium use for electricity generation are explored with a linear programming model (ALPS) of nuclear activities in the United States extending into the next century (Manne *et al.*, 1979). For many scenarios covering exogenous electricity demands, and costs for capital, fuel, and operations, the minimum-cost nuclear activities are determined. The long time horizon, and the representation of uranium availability, cover depletion effects to an extent.

A broader, linear programming approach is taken in the dynamic energy sector model DESOM (Hoffman and Jorgenson, 1977). Energy resources, technology and end-use demands are covered with scenarios, and the minimum cost of operating the whole energy sector, so as to meet these demands, is sought. Resource availability constraints introduce depletion effects. A similar approach, widened to cover country-to-country variation and hence trade, is employed in empirical investigation of the world energy situation (Nordhaus, 1973).

Price elastic demand assumptions are employed in a largely linear intertemporal energy sector model (ETA), constructed to examine long-term technological change (Manne, 1976). The non-linear objective can be interpreted as the discounted sum of consumer plus producer surplus, and is maximized subject to technological and resource constraints, which introduce depletion effects.

An efficient solution algorithm for price sensitive depletion models has been developed (Modiano and Shapiro, 1980). A resource-directed decomposition scheme allocates the non-renewable resource across time periods, solves a 'value of supply' problem in each time period to derive the marginal value of the resource, reallocates the resource, and so on in a convergent sequence to the optimal resource use. The approach is applied to the U.S. coal sector, and employs an interesting resource description.

This is a set of long-run supply curves for coals of various types and locations, which constitutes an economic resource endowment measure (Zimmerman, 1977). Geological data and the input requirements for mines are combined to form extracted-cost/quantity relationships. These are sorted to produce approximate long-run supply curves. Economic endowment measures have been called for in discussions of required theoretical advances (Smith, 1980; Harris and Skinner, 1982; Bohi and Toman, 1984).

The price-sensitive depletion model has been extended to the uncertain-substitute-arrival context. The exhaustion of low-cost uranium while "Waiting for the Breeder" is investigated with a probabilistic linear program (Manne, 1974). This is structured like a decision-tree; at several dates in the future the reactor may arrive, or not, with 'known' probability. The surplus maximizing outcomes have low sensitivity to uncertainty for the range of probabilities and discount rates examined. The expected value of perfect information is low for this situation (Chao, 1981).

3.5.2 National models

The ETA model can be linked to a macroeconomic production function which represents the rest of the economy, to form ETA-MACRO. In this approximate general equilibrium model, rising energy costs and dwindling resource availability affect the growth rate and hence the energy demand curve (Hudson and Jorgensen, 1974). A further extension embeds ETA-MACRO in a decision-tree framework covering breeder-reactor program uncertainties. Depletion effects are captured to some extent by the cumulative resource cost functions and long horizon.

A different approach to forming a national depletion model is to add a non-renewable resource characterization to a macro-econometric model (Motamen, 1983). This investigation of the depletion of U.K. North Sea oil reserves takes an optimal control approach: the extraction rate and domestic and foreign investment rates are chosen so as to maximize 'national wealth'. The optimal control settings are numerically approximated for each deterministic scenario as to world oil prices and other parameters. The optimal depletion rate is sensitive to world oil prices.

3.5.3 Discussion

The modelling exercises above apply existing theories to more realistic situations. Discounted utilitarian objectives, usually based on consumer surplus, are used. Risk-neutral attitudes to uncertainty are adopted. The solution is always a precommitment plan or strategy.

The exercises do not widen the theoretical base for principles guiding non-renewable resource use. The exercises have a mostly unrealized potential to test the relevance and sensitivity of existing theory to more realistic assumptions. The exercises also suggest that numerical exploration of theoretical issues is feasible: objectives and attitudes to risk which are otherwise analytically intractable could be numerically explored.

3.6 Conclusions

Theoretical investigations of many kinds contribute to the development of principles which guide decisions about non-renewable resource activities. This chapter has reviewed the theoretical economic investigations which are directly concerned with these principles for uncertain contexts. Many shortcomings of these investigations are discussed above, and more general limitations of the economic approach to uncertainty are discussed in Chapter Two. An overview of the shortcomings, and an elaboration on some of them, is now presented, emphasizing those with particular importance for non-renewable resource use. The chief limitations are that:

- extreme simplifications of institutional structures and technology are employed,
- a narrow range of ethical positions and objectives are examined,
- precommitment strategies are always derived, so the position in time of decision-making is poorly represented.

Non-renewable resource actions are in reality strongly influenced by the prevailing institutional structure, comprising the system of property rights, the market power of the participants, and the general policy environment including the taxation structure. Institutional structures are either extremely simplified, or ignored entirely, in the resource investigations. The descriptive analysis generally assumes competitive resource ownership and demand, with the exception being the treatment of strategic behaviour in substitute development. The prescriptive investigations are sometimes presented as problems for a societal planner or a command economy, but only physical attributes of the resource are modelled.

Some aspects of the neglect of the institutional structure are more important for non-renewable resources than for other issues. Institutions which help shape beliefs about the future resource situation are among these aspects. Central and decentralized ways of providing individuals with good information about resource possibilities, substitute possibilities, and demand possibilities are important, but are outside the investigations. The **impact** on resource use of expectations and of contingent markets has been discussed. Whether and how these influences are to be improved or impeded has not been discussed. Most of the descriptive investigations' conclusions are called into question because they rely on the presence of institutions which are not available.

The description of the resource base and the technologies for resource extraction, conversion, and use are extreme simplifications. For example, resource use inevitably results in waste creation, which may be costly or detract significantly from welfare, but which is ignored in the investigations. The sensitivity of the conclusions, to these simplifications, is unknown. The partial equilibrium assumption assumes a limited dependence between the resource sector and other activities. There is no allowance that resource activities may affect income and shift resource demand curves in 'positive' models. In prescriptive models the resource sector is totally independent, which seems untenable. The conclusions may therefore not extend to situations with realistic dependencies between activities. Similarly, interdependencies between environmental influences and human activities may be important, but are ignored.

The resource investigations employ or are consistent with a narrow range of ethical positions/objectives. The 'positive' investigations focus on *ex-ante* efficiency. As Chapter Two discusses, *ex-post* efficiency may be an equally compelling societal objective. Its achievement, or otherwise, when there are non-renewable resources and uncertainty, is unknown.

Two prescriptive investigations consider Rawlsian maximin objectives. Expected discounted utilitarian objectives are otherwise used, and the rationale for discounting is rarely stated. It can be justified on grounds that other sectors are providing increased consumption, or that there is a chance of 'Armageddon'. These grounds are quite different from discounting as an ethical position, and deserve exposure.

The dynamic, uncertain context of the non-renewable resource use issue severely tests the economic approaches. The 'positive' approaches largely side-step the issue by examining only *ex-ante* criteria. It seems likely that much of the following discussion would be pertinent to 'positive' examinations of *ex-post* objectives.

The prescriptive investigations are not clear about the decision-maker's position **in** time. Optimal plans or strategies cover all time periods, but are optimal with respect to one objective only. This objective covers all time periods, so must be interpreted as the objective of the **initial** time periods' decision-maker.

The choices of later decision-makers are not explicitly incorporated via their objectives, or via constraints on the later possibilities. The optimal strategies produced as solutions must therefore be optimal **precommitment** strategies; i.e. strategies which are optimal conditional on their being **followed** in later periods, although **derived** in the first. Such a strategy must either be able to be imposed on later decision-makers, or be a strategy all later decision-makers would voluntarily adopt, to make sense as a solution.

If neither property holds, the strategy is inconsistent as an optimum set of actions: initial actions are optimal conditional on the later 'optimal' actions, but these are not expected to be followed, so the initial actions cannot be optimal. That is, the strategy only appears optimal to the initial decision-maker because of the systematic mistake of thinking that later actions in the strategy will be followed. This wishful thinking is not compatible with a rational approach to identifying appropriate actions.

There appears to be no way of forcing later decision-makers to adopt set actions, so a precommitment strategy is only justifiable if it is (believed to be) adopted voluntarily by all later decision-makers. This effectively assumes that the different decision-makers' orderings, of continuations of the program of actions, are the same. Equivalently, decision-makers objectives can differ only in ways which do not lead to a difference in choice.

This 'same ordering' assumption is very strong given the long times involved in non-renewable resource investigations, and casual observation of changing real objectives. The assumption underpins the 'expected integral of discounted utilities' objective used throughout the investigations. It appears this objective is used largely **because** it can be interpreted as fulfilling the 'same ordering' assumption: a precommitment strategy which is optimal for this objective is optimal **without** wishful thinking, for one imaginable sequence of decision-makers. The sensitivity of the conclusions to a change in this sequence has not been investigated to date.

The reliance on the utilitarian objective is reinforced by limiting the concept of a solution: precommitment strategies are derived in **all** investigations of non-renewable resources and uncertainty to date. These strategies are found with the analytically tractable techniques of stochastic dynamic programming and stochastic control, which admit only one objective.

However, analytical convenience is not a justifiable reason for restricting investigations to precommitment strategies when this restricts the sets of objectives which can be examined. There is a need for analytical procedures which derive consistent (no wishful thinking) sets of optimal actions for arbitrary sets of objectives (differing orders).

The concentration on precommitment strategies and their associated analytical techniques may 'explain' other restrictions in the range of investigations performed. Only one investigation applies a non-technological constraint in derivation of an optimum solution (Riley, 1980), although society might be expected to rule out many actions as being unethical *a priori*. Only one investigation employs an unobservable state variable (Battacharya, 1984), although in general the state is a 'state of belief', best represented by a probability distribution or stochastic process.

Precommitment strategies may also obscure the fundamental purpose of the investigation: to generate insight or principles which guide actions, which are always taken in the here-and-now. The relationship between current beliefs (about the future and 'the good') and appropriate current actions is not the clear focus of a precommitment strategy, because this consists of an optimal action for each future circumstance. In Chapter Four recursive decision programs which avoid the consistency issue, allow investigation of arbitrary sequences of objectives, and focus on initial actions, are developed.

Chapter Four

Recursive Decision Models

The chapters above show that existing economic approaches for investigating non-renewable resource use have serious limitations. The dynamic, uncertain nature of the issue is poorly dealt with. The position **in** time of the decision-maker is not well understood.

These limitations will persist while precommitment strategies are employed as solutions. These overstate the initial decision-maker's power to determine the course of events. The overstatement seems likely to be most seriously misleading where actions have traceable long term consequences so the eventual **sequence** of decision-makers is likely to have varying concerns.

Non-renewable resource use is a case in point; the impact of future changes in concerns, on current 'best' resource use, cannot be explored with existing approaches. The concentration on precommitment strategies limits the investigations in other ways as well. Many possible attitudes towards future wellbeing cannot be explored with precommitment approaches, because the associated 'solutions' are intertemporally inconsistent and therefore have little meaning. Attitudes towards risks of various sorts, at various times, are particularly circumscribed by the precommitment approach.

In this chapter a recursive decision approach is developed, so as to widen the investigation of non-renewable resource use. The deeper understanding of resource use issues that this approach potentially offers may enable decisions influencing resource use to be made in a more informed way.

The approach developed here extends the decision-theoretic model of choice. The 'solution' sought is a set of actions consistent with the choices of a **sequence** of decision-makers. This requires that each decision-maker fixes their **commitment** actions only.
Later decisions about action, whether undertaken by a later version of the initial decision-maker, or by later different decision-makers, are among the **consequences** of initial choice, and must be **forecast** not predetermined. The forecast of later actions needs to take account of how later decisions might be made by later decision-makers. This in turn requires a forecast of later decision-makers' forecast and preferences.

The structure of the recursive decision model is first developed heuristically as an extension of a decision-theoretic model, so as to emphasize where the structures differ. A more rigorous discrete formulation of the recursive structure is then developed, so that the following discussion can be more precise. The existence of 'solutions' for the discrete model and for feasible extensions, and the assumptions implicit in the structure are then dealt with in turn. The chapter concludes with a discussion of the advantages and disadvantages of the recursive decision approach.

4.1 Heuristic development

This section heuristically introduces the key features of a recursive decision model. If these features are not explicitly present, in a prescriptive investigation of a dynamic uncertain situation, then the implications of the investigation are either mistaken or are based on implicit assumptions. The well-known decision-tree model is gradually modified until all the features are present.

4.1.1 Decision trees

In an evolutionary system, actions occur at different times and uncertainties about what will occur are gradually resolved. Let there be a set $T = \{1,2,3...\}$ of time periods t: at the beginning of each time t an action a_t is selected from a finite set A_t. For the moment A_t is independent of the history of the system.

Some uncertainty is resolved instantaneously immediately after the decision, when s_t, one of a finite set S_t of possible uncertain events, is observed to occur. S_t is for now independent of the system history.

The possible futures form a decision tree structure as in Figure 4.1, which takes $A_1 = \{A,B\}$, $A_2 = \{C,D,E\}$, $S_1 = \{1,2\}$, $S_2 = \{3,4\}$. The horizontal lines depict the passage of real time, while the 'branching out', representing actions and events, occurs 'instantaneously' at the start of each period. Each possible future is one path through the tree, that is, a sequence of actions and events of form $\{a_1,s_1,a_2,s_2,...\}$. There are 24 possible futures for the two-period time span covered in Figure 4.1, between $\{A,1,C,3,...\}$ at the top and $\{B,2,E,4,...\}$ at the bottom. The letters and numbers on the branches of the tree are labels, not measurements of any underlying property.

During each time period a state of affairs q_t, from the possible set Q_t, prevails. Each q_t is uniquely determined by the preceding actions and events. The set of possible futures is the set of **sequences** of actions and outcomes which can be formed.

When A_t and S_t are independent of the history then H_t, the set of histories h_t which might be observed by the end of time t, is defined as:

$$H_t \equiv \left[\langle a_i, s_i \rangle_{i=1,2,3...t} \;\middle|\; \begin{array}{ll} a_i \in A_i & \text{all } i = 1,2,3...t \\ s_i \in S_i & \text{all } i = 1,2,3...t \end{array} \right]$$

By definition, $q_t = f_t(h_t)$.

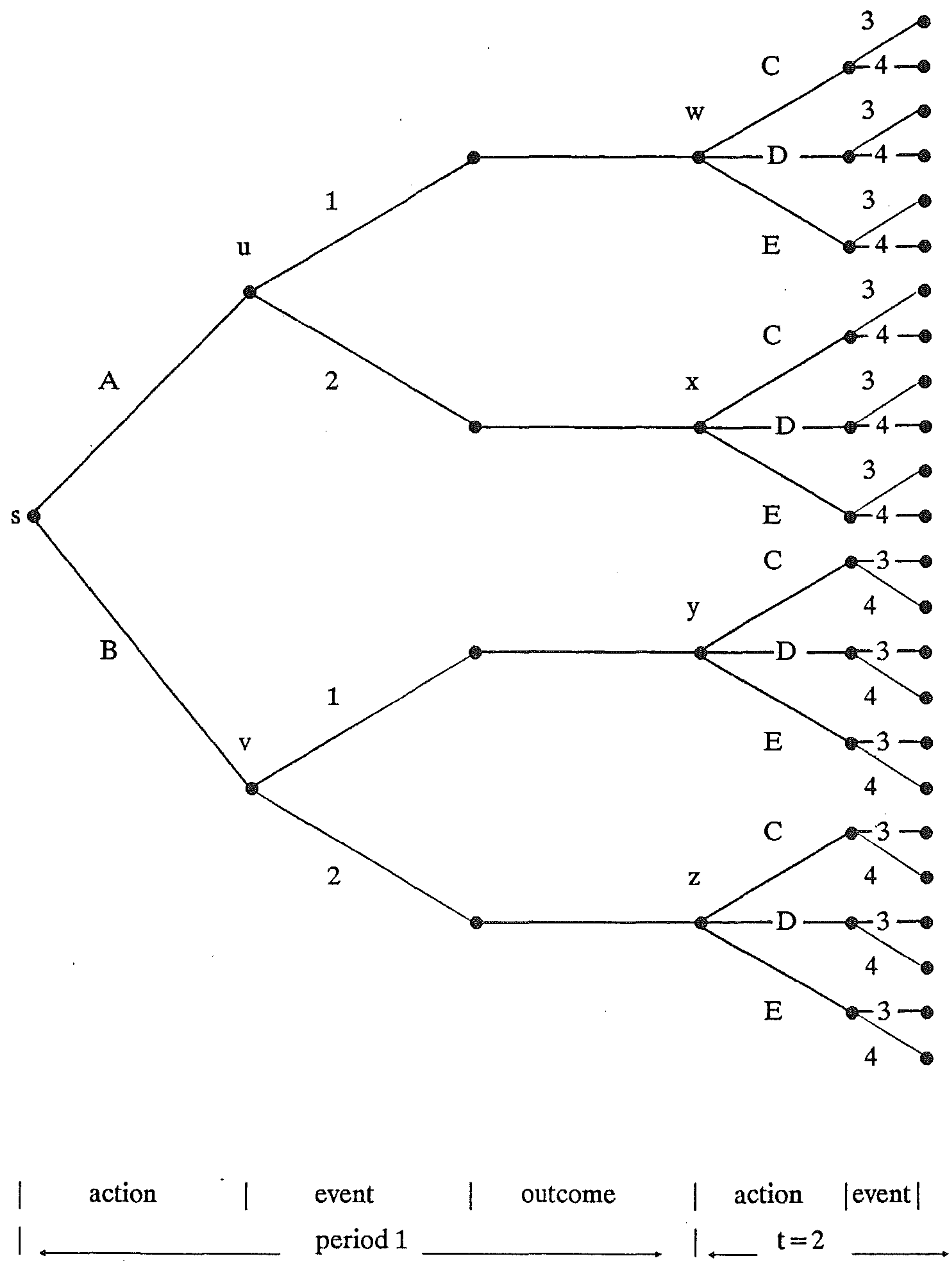

Figure 4.1: A tree of possible futures

The actions are the subject of inquiry and are chosen. Known (subjective) probabilities govern the possible events. The probabilities $p_t(s_t)$ of the possible events at each time sum to one:

$$\sum_{s_t \in S_t} p_t(s_t) = 1, \quad \text{all } t \in T.$$

No probabilities are specified in the figures in this section, but they are assumed to be available for each uncertainty resolution illustrated.

A strategy is a set of actions, one for each decision node in the tree. Therefore, a strategy contains an action for each possible history of the system, up to each time. The explicit tree structure helps the decision-maker identify the preferred strategy - the one with the preferred actions, events and outcomes.

All actions are chosen by the one decision-maker. Usually, the assumed measure of success enables this to be done by backwards recursion, in accordance with the "Principle of Optimality" or "Maximum Principle" which underpins the analytical methods of dynamic programming (Bellman, 1957) and optimal control theory (Pontryagin *et al.*, 1962). This states that in any optimal strategy, regardless of how a history comes about, the decisions from then on must be made optimally.

In backwards recursion a terminal time T^f is set first, perhaps because nothing after this time matters to the decision-maker. The preferred action at time T^f-1 is found then for every history $h_{T^f-1} \in H_{T^f-1}$. If in Figure 4.1 T^f is set to two, the first decisions addressed are at w,x,y and z. These decisions take into account only one remaining resolution as to events. Each history's optimal actions at T^f-1, and value of the program after that date, are then regarded as fixed.

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The procedure then examines the decisions at T^f-2, on the basis of the T^f-2 variables and the T^f-1 fixed values. As the procedure continues backwards through time, each examination deals with only one resolution of uncertainty. When the optimal action for the initial decision in the tree is found, the complete optimal strategy is available. Under the optimal strategy, most of the branches in the tree are no longer possibilities.

The assumption that A_t and S_t are independent of h_{t-1} is generally untenable, and the backwards recursion solution method is not limited to this case. Redefining, let $A_t(h_{t-1})$ be the set of possible actions given a history $h_{t-1} \in H_{t-1}$ (to be redefined accordingly below).

The set J_t of paths j_t preceding a resolution at time t is:

$$J_t = [\, j_t = \langle h_{t-1}, a_t \rangle \mid h_{t-1} \in H_{t-1}, \;\; a_t \in A_t(h_{t-1}) \,], \;\; \text{all } t \in T$$

The resolution's dependence on j_t is captured by defining the possible events to be members of the finite set $S_t(j_t)$. The conditional probability of an event s_t, given a history j_t is defined to be $P^R(s_t, j_t)$, so that,

$$\sum_{s_t \in S_t(j_t)} P^R(s_t, j_t) = 1, \;\; \text{all } j_t \in J_t, \;\; \text{all } t \in T.$$

In Figure 4.1, for example, $H_1 = \{(A,1),(A,2),(B,1),(B,2)\}$ and $J_2 = \{(A,1,C),(A,1,D),(A,1,E),(A,2,C),\ldots(B,2,E)\}$, $A_2(A,2) = \{C,D,E\}$ and $S_1((B)) = \{1,2\}$.

Redefining H_t to account for dependence, let $B = \{1,2,3\ldots t\}$:

$$H = \{ \langle a_i,s_i \rangle_{i \in B} \mid a_1 \in A_1; \;\; s_i \in S_i\,(a_1,s_1,a_2,s_2,\ldots a_i) \quad \text{all } i \in B;$$
$$a_i \in A_i\,(a_1,s_1,a_2,s_2,\ldots s_{i-1}) \quad \text{all } i \in B \}.$$

In approximating 'reality', the decision tree has each resolution of uncertainty occurring instantaneously immediately after a decision. If in reality the resolution (and the decision) occurs gradually **throughout** the time period, an equally valid approximation model places the whole resolution immediately **before** the decision at the start of the period.

The former structure is 'under-anticipative', or pessimistic since at the decision time none of the 'period's-worth' of uncertainty resolution is known. The latter is 'over-anticipative', or optimistic since at the decision time the whole 'period's-worth' of uncertainty resolution is known. With long time periods the strategies derived from either structure may be sub-optimal. The problem disappears with continuous time techniques, such as stochastic control and some forms of stochastic dynamic programming.

4.1.2 Rolling planning

In a rolling planning scheme an optimal plan or strategy covering several time periods is derived, perhaps with a decision tree, but only the first period's optimal action is implemented. Next period, a whole new decision tree and optimal strategy are derived and the first period's action is implemented. Each new decision tree takes account of new beliefs, gathered in the course of the period just passed.

Any decision-tree must be intended to help some particular decision-maker. Humans exist **in** time; they cannot adopt the view available to an hypothetical 'extratemporal' observer. A decision-tree therefore applies at a point in time, and only influences decisions not yet made, so the structure is constructed by or on behalf of the entity charged with making the **first** decision, for the primary purpose of helping with that decision.

Logically, only the initial (present) actions can presently be undertaken, so only initial decisions can presently be implemented. Future decisions captured in present optimal plans or strategies are reconsidered in new trees when these decisions have, in their turn, become the initial decision. The present optimal strategy may provide a starting place in this later examination.

The extent to which each new tree differs from the last depends on how well understood the processes of change are. The new tree may be essentially a reaffirmation of a previous sub-tree, or it may be radically different. New trees cannot always be 'completely different', if they do in fact amount to worthwhile forecasts. To the extent that human beliefs capture reality, the tree informs the **initial** decision-maker.

4.1.3 Choice versus forecast

The actions which can be implemented by the initial decision-maker are termed **commitments**, to distinguish them from later actions termed **recourse**. In disagreement with the decision-tree view, since recourse cannot be presently implemented, recourse must be **forecast**, and not chosen, by the initial decision-maker.

This position is the only one consistent with locating the whole decision-tree at a point in time as an analytical tool of the decision-maker at that time. If recourse actions do not need to be forecast this can only be because they can be committed-to at the initial time, in which case they are not in the recourse set: the idea of recourse which does not need to be forecast is internally inconsistent.

Depending on the time interval involved, the decisions about recourse might be expected to be made by a later version of the initial decision-maker, or made by someone else. If the initial decision-maker expects to remain the decision-maker the best forecast of the recourse choice may well centre on the initial decision-maker's current choice. The best forecast might then be to 'choose as the initial decision-maker would', so that the decision-tree structure is recovered.

In general, there is uncertainty about which choice procedure v_t from a possible set V_t will be applied at time t. This is captured by augmenting the uncertainties in the tree structure as shown in Figure 4.2. Here it is initially thought that choice procedure G_1 or G_2 will be applied in the second period.

Maintaining the under-anticipative formulation, the uncertainty about v_{t+1} is resolved instantaneously at the end of period t, just before a_{t+1} is decided on. In Figure 4.2 uncertainty about recourse is independent of the history, but this is generally untenable.

The new augmented structure is a recursive decision model. Redefine the set H_t of histories h_t which are *a priori* observable before the end of time t:

$$H_t = \left[\begin{array}{ll} h_t = \langle a_i, s_i, v_{i+1} \rangle_{i \in B} \; | a_1 \in A_1; & \\ \quad s_i \in S_i(a_1, s_1, v_2, a_2, s_2, \ldots a_i) & \text{all } i \in B; \\ \quad v_i \in V_i(a_1, s_1, v_2, a_2, s_2, \ldots s_{i-1}) & \text{all } i \in B,\ i \neq 1; \\ \quad a_i \in A_i(a_1, s_1, v_2, a_2, s_2, \ldots v_i) & \text{all } i \in B,\ i \neq 1; \end{array} \right]$$

where $V_t(k_{t-1})$ is the set of choice procedures which might be adopted at t given history $k_{t-1} = (h_{t-2},\ a_{t-1},\ s_{t-1})$, drawn from the possible set K_{t-1}.

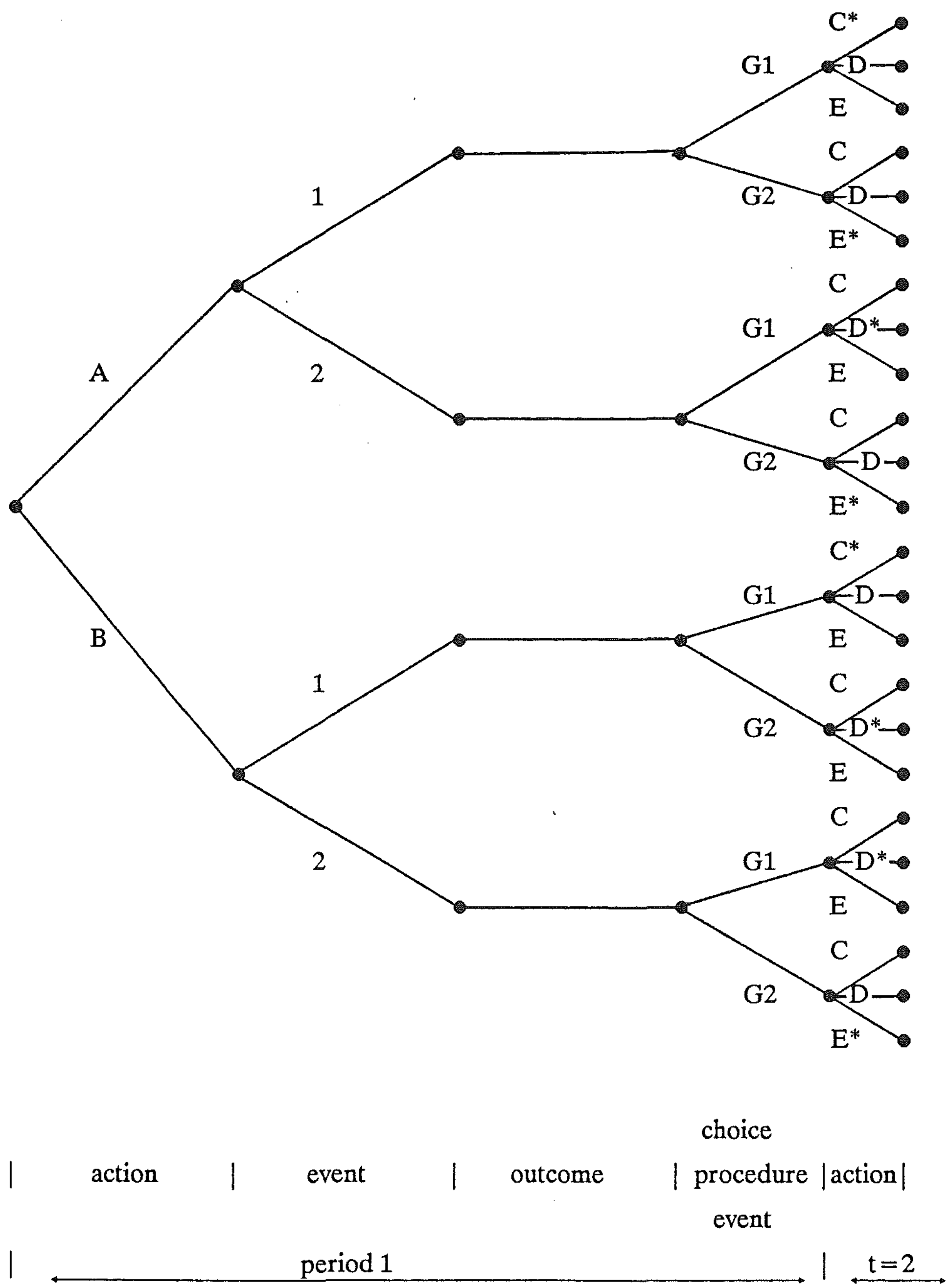


Figure 4.2: Forecasting later choice procedures

The probability of v_t, conditional on k_{t-1}, is $P^W(v_t,k_{t-1})$, so that

$$\sum_{v_t \in V_t(k_{t-1})} P^W(v_t, k_{t-1}) = 1, \qquad \text{all } k_{t-1} \in K_{t-1}, \text{ all } t = (2,3,4,..).$$

In Figure 4.2, for example, $K_1 = \{(A,1),(A,2),(B,1),(B,2)\}$ and $V_2(B,1) = \{G_1,G_2\}$.

It may be necessary to augment the tree structure to cover actions, events and outcomes which are important in recourse decisions, although not directly important to the initial decision-maker. Such happenings may affect later actions and so indirectly become important to the initial decision-maker.

Further, the tree may need to be augmented to reflect initial beliefs about later beliefs. By assumption here, the tree represents the knowledge of a fully 'rational' decision-maker, which is taken to mean that the subjective probabilities placed on all future occurrences conform to Bayesian standards.

If the initial decision-maker believes later persons are similarly rational, then no augmentation to cover future beliefs is necessary, because it must be believed that future beliefs will amount to one or another subtree of present beliefs. Augmentation is necessary if it is believed that future persons may be 'irrational', or that their arbitrary judgements about irreducible uncertainties may differ from present arbitrary judgements.

Thinning

If all forecast later choice procedures are sufficiently well-defined and a terminal time T^f is specified, the information now in the tree eliminates all decisions except the first. By backward recursion from T^f, for each history h_t, the associated v_{t+1} (and not initial preferences) selects an action a_{t+1}, on the basis of the probabilistic tree of consequences.

In Figure 4.2 illustrative selected actions for each h_1 are starred. The resulting 'thinned' tree in Figure 4.3 demonstrates that each initial choice leads to a sub-tree branching only on uncertain events. Each sub-tree is the initial decision-maker's full forecast of future actions and events, conditional on an initial action. Only the initial decision is still to be considered.

4.1.4 Choice procedures

A rational decision-maker determines which action to take on the basis of some or all of the information in a thinned tree, by applying a choice procedure to this information.

The procedure works by elimination of all but one of the possible actions. Elimination may take several stages, but must eventually identify a single 'best' action. **Consequentialist** choice procedures identify appropriate actions on the basis of any aspect of the envisaged tree **excluding** the initial actions themselves; these procedures are concerned with what is brought-about by action, and commonly concentrate on the possible states of affairs as measured by q_t. However, the initial actions themselves are within the tree, and may be arguments in choice procedures.

Elimination of actions takes place in two ways. An action may be rejected because it does not meet a standard the decision-maker believes in. Standards may be absolute, as when certain types of action are judged to be wrong in themselves, or relative, as when an action does not fall in (say) the top half of the set on a measure of performance. Alternatively, the optimization of a measure of performance eliminates all actions except the optimizing action.

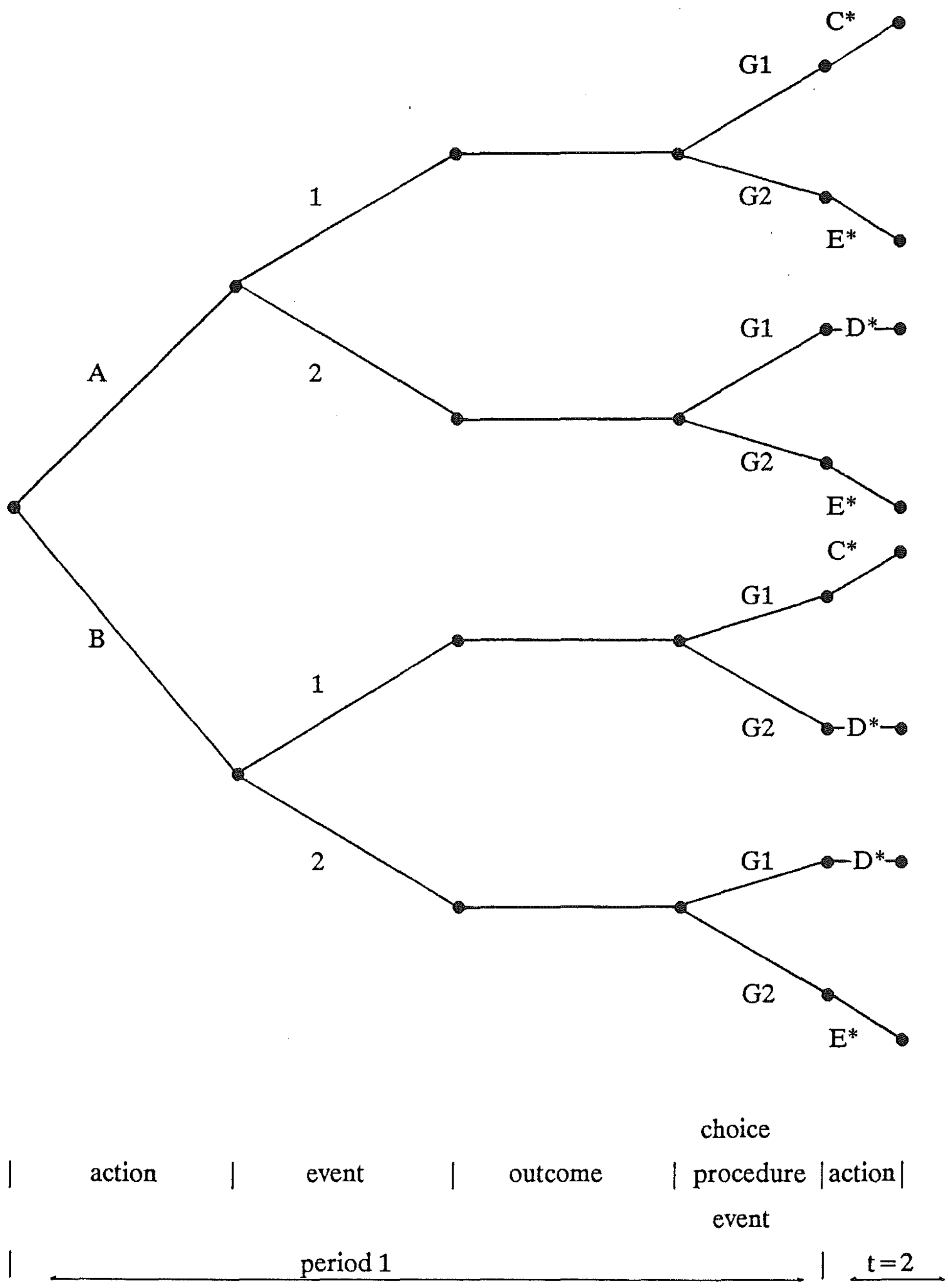


Figure 4.3: A thinned tree

Because one action **must** be adopted, the choice procedure must include a back-tracking routine to identify an appropriate action when no action can meet an absolute standard. The procedure must also be decisive, and select **one** action. A final **arbitrary** selection, from a set of appropriate actions, may be treated by replicating the following tree once for each appropriate action, and adding a uniform distribution on corresponding 'dummy' choice procedures to the preceding choice procedure possibilities.

As outlined in Chapter Two, the choice procedure usually employed in economic theorizing is to maximize the subjective expected utility (SEU) of consequences. When the consequences are further tree structures of uncertainty resolution, the utility measure is generally applied to paths within the tree rather than to sub-trees. A utility indicator $U(q_1,q_2,q_3,...)$ is adopted, a terminal time Z is specified, and the set H_Z of paths h_Z in the thinned tree is identified. Denote by $H_Z(a_1)$ the subset of H_Z containing $a_1 \in A_1$. The probability $P^H{}_Z$ of $h'_Z \in H_Z(a_1)$, which is only defined if a_1 is chosen, is the product of the probabilities of the resolutions in h'_Z:

$$P^H{}_Z(h'_Z) = \prod_{t=1}^{Z} P^R(s'_t, j'_t) \times P^W(v'_{t+1}, k'_t), \quad \text{all } h'_Z \in H_Z(a_1),$$

where if

$$h'_Z = (a_1, s'_1, v'_2, a'_2, s'_2, ... v'_Z, a'_Z, s'_Z, v'_{Z+1})$$

then

$$k'_t = (a_1, s'_1, v'_2, a'_2, s'_2, ... v'_t, a'_t, s'_t)$$

and

$$j'_t = (a_1, s'_1, v'_2, a'_2, s'_2, ... v'_t, a'_t).$$

Also, the sequence of consequences occurring with h'_Z is

$$< q'_t = f_t(k'_t) > \quad \text{for all } t = (1,2,...Z).$$

Therefore, letting $U(h'_Z) = U(q'_1, q'_2, \ldots q'_Z)$, the expected utility of an action a_1 is:

$$\sum_{h_Z \in H_Z(a_1)} P^H{}_Z(h_Z) \times U(h_Z).$$

The action $a_1^* \in A_1$ which maximizes this measure is to be adopted, according to the usual presentation of SEU choice.

This procedure takes no direct account of much of the information in the tree. A decision-maker can, consistent with the axioms of SEU, include as arguments of the utility indicator any aspect of the thinned tree except the initial action a_1 and the initial event s_1.

As envisaged, the commitment is followed by a time stream of **situations**, each of which involves a future person contemplating an approaching decision. The initial decision-maker may believe the experience of unresolved uncertainty in those situations is relevant to initial choice, irrespective of whether those uncertainties 'turn out well' or not: this belief cannot be acted on in the usual SEU version.

The future choice procedures v_t may concern the initial decision-maker: he/she may be more interested in bringing about 'morally sound future persons' than in bringing about 'wealthy' material states of affairs. This may still be true when the decision-maker envisages continuing to make decisions; the initial concern may be to avoid evolving into a user of those v_t which are currently seen as 'corrupt', even though it is envisaged that the choices made would be enjoyed.

4.1.5 Irreducible unknowns

Some uncertainties about the future can only be represented by probability measures if these are arbitrary, that is not based on observations and theories. Given this, a **set** of thinned trees would better represent the ramifications of action, where adopting one tree over another would require purely arbitrary judgements. Therefore, for each initial choice procedure there may be a **set** of appropriate initial actions; one initial action for each thinned tree. To decide that one of these actions is best because its tree is most likely is a purely arbitrary judgement.

The decision-maker can be more fully informed by considering **how** appropriate each action is in each tree. Further, arbitrary likelihoods of the trees sum to one if it is maintained that only one tree is 'true'. If the choice procedure is extended to cover an imaginary initial resolution of the irreducible uncertainty, the 'appropriate' actions for each arbitrary judgement can be found.

Similarly, the set of arbitrary judgements which indicate that each action is appropriate can be found. Some actions may thereby be eliminated because they are not appropriate for any arbitrary judgement. Exposure of the interdependence of beliefs and appropriate actions in this manner is the most that can be done to inform the decision-maker about the merits of action, for each given choice procedure.

There is no single morally compelling choice procedure. Adoption of a choice procedure is therefore arbitrary to some extent, and there is considerable symmetry between this arbitrariness and the arbitrariness in uncertainty judgements. Therefore, for each thinned tree there may be a set of actions which are indicated to be appropriate; one action for each of the set of justifiable choice procedures.

The decision-maker can be more fully informed by considering how appropriate each action is for each justifiable choice procedure. For measurable choice procedures it is possible to derive the appropriate action for each possible weighting over procedures. Also, the set of weightings over procedures which is consistent with adopting an action may be identifiable. Some actions may be eliminated because they are not appropriate for any weighting over the choice procedures.

As with irreducible uncertainties, exposure of the interdependence of beliefs and appropriate actions is the most that can be done to inform the decision-maker about the merits of action, for each tree. Recognizing **both** forms of limit requires that an informed decision-maker is aware of the merits of each action as this is judged by each choice procedure for each tree.

4.1.6 Features of a sound approach to dynamic choice

Any model from which principles are to be drawn to guide action must distinguish between initial actions (commitments) which can be implemented at the initial time, and later actions (recourse) which may be chosen later. The later choices must be forecast, which may require forecasting choice procedures.

Each tenable forecast must be a thinned tree structure of gradual uncertainty resolution over time. The subjective probabilities of all events must be Bayesian consistent. A set of such trees is required to reflect the irreducible uncertainties.

Each justifiable choice procedure must have as its arguments aspects of actions and their consequences which are drawn from a thinned tree, so as to eliminate wishful thinking. A set of choice procedures is required to reflect the non-existence of a single morally compelling procedure.

A fully informed decision-maker is aware of the dependence of appropriate actions on the beliefs represented by the set of thinned trees and the set of justifiable choice procedures.

4.2 Recursive trees

A formal representation of the recursive decision approach is now developed, to allow more precise discussion of its features. The situation the formal model approximates is one where each decision-maker makes a commitment to one of a number of actions. Before the commitment, the decision-maker believes the consequences of each action are best represented as a stochastic process with known parameters.

The possible evolutions of this process indicate which future decision-makers, actions, and events are believed to be possible, and what their relative likelihoods are believed to be. The decision-maker believes future decision-makers will believe likewise, so the imagined consequences are recursive. The model developed here employs **discrete** time, actions, events, and choice procedures to represent a fully recursive belief structure.

4.2.1 Branches and nodes

Time consists of a number of discrete periods indexed $t = 1,2,3....$ Let a sequence of happenings before period t be temporarily denoted h_t. At the beginning of each period a decision-maker selects and undertakes an action a_t from a finite set $A_t(h_t)$ which depends on the history h_t. Following this, a 'random' event s_t occurs, from a finite set $S_t(h_t,a_t)$ which depends on the history.

A set of circumstances q_t from the set Q_t then prevails until just before the next period; q_t may be a function f_t on a_t, s_t, and h_t. At the end of the period a further 'random' event v_t from a finite set $V_t(h_t,a_t,s_t)$ resolves the uncertainty about the next decision-maker's choice procedure. V_t may depend on previous happenings. Note that v_t in this construction affects choice in $t+1$.

The points of decision about action, and of resolution as to events, are the nodes of a recursive tree. At each decision node selection of a branch a_t is equivalent to selection of the next resolution node. At each resolution node occurrence of the branch s_t or v_t amounts to a 'choice by nature' of the next resolution or decision node.

Let the set of decision nodes be D, generic member $d \in D$, the set of exogenous resolution nodes be R, generic member $r \in R$, and the set of choice procedure resolution nodes be W, generic member $w \in W$. Define the total node set $N = D \cup R \cup W$, with generic member n. Let there be a unique initial node $n_0 \in D$.

Define D(n), R(n) and W(n) as the sets of decision, exogenous resolution and choice procedure resolution nodes following n in the tree, augmented to include n if n is of the respective type. This defines $N(n) = D(n) \cup R(n) \cup W(n)$.

It is fundamental to the tree structure that branches do not rejoin, so if

$$n^1 \in N(n^2) \text{ then } N(n^1) \subseteq N(n^2).$$

Also, if $n^1, n^2 \in N(n^3)$ then either

$$N(n^1) \subset N(n^2) \text{ or } N(n^2) \subset N(n^1) \text{ or } N(n^1) \cap N(n^2) = \varnothing.$$

Let, $D^f(w)$ be the set of decision nodes immediately following choice procedure resolution node w, $R^f(d)$ be the set of 'exogenous event' resolution nodes immediately following node d, and $W^f(r)$ be the set of choice procedure resolution nodes immediately following node r. $N^f(n)$ is the set of resolution nodes **or** decision nodes which immediately follow node n, depending on the type of n. The decision at each d is to choose a member $r \in R^f(d)$, the resolution at each r amounts to 'selection' of one $w \in W^f(r)$, and the resolution at each w amounts to 'selection' of one $d \in D^f(w)$.

The collection:

$$<N(n)>_{n \in N}$$

represents the whole tree structure.

4.2.2 Sequences and strategies

For the time being the impact of v_t on choice in period $t+1$ is ignored. Histories and possibilities for the system can be equally well expressed as sequences of nodes or of branches. A branch is now identified with the node it leads to.

The conceivable infinite sequences of actions a_t form a set A with generic member a:

$$A = \left[\begin{array}{l} a = <a^*_t>_{t=1,2,3...} \mid \text{for all } t = 1,2,3... \\ \qquad \text{there exist } w^* \in W \text{ and } d^* \in D, \text{ such that:} \\ \qquad w^* \in W^f(a_t),\ d^* \in D^f(w^*),\ a^*_{t+1} \in R^f(d^*) \end{array} \right]$$

The infinite sequences of random events s_t, v_t form a set S, with generic member s:

$$S = \left[s = <s^*_t, v^*_t>_{t=1,2,3...} \;\middle|\; \begin{array}{l} s^*_1 \in W^f(r^*) \text{ for some } r^* \in R^f(n_0) \\ s^*_t \in W^f(r^*) \\ \qquad \text{for some } r^* \in W^f(v^*_{t-1}), \\ \qquad \text{for all } t = 2,3,4... \\ w^*_t \in D^f(s^*_t) \text{ for all } t = 1,2,3... \end{array} \right]$$

The overall histories form the set:

$$H = \left[h = \langle h^a_t, h^s_t, h^v_t \rangle_{t=1,2,3...} \;\middle|\; \begin{array}{ll} h^a_1 \in R^f(n_0) & \\ h^a_t \in R^f(h^v_{t-1}) & \text{for all } t=2,3,4... \\ h^s_t \in W^f(h^a_t) & \text{for all } t=1,2,3... \\ h^v_t \in D^f(h^s_t) & \text{for all } t=1,2,3... \end{array} \right]$$

At each node n the preceding actions and events are given; the set A(n) is defined to contain all sequences of action which are still feasible at n, so that each sequence $a \in A(n)$ contains the fixed actions preceding n. S(n) is the set of sequences of events which are still possible as at node n, so each sequence $s \in S(n)$ contains the events which preceded n. Each history $h \in H(n)$ contains the actions **and** events which precede n. Formally:

$$A(n) = \left[a^* \in A \;\middle|\; \begin{array}{ll} \text{if } n \in R \text{ then } a^*_t = n & \text{for some } t=1,2,3... \\ \text{if } n \in D \text{ then } a^*_t \in R^f(n) & \text{for some } t=1,2,3... \\ \text{if } n \in W \text{ then } a^*_t \in R^f(d^*) & \text{for some } d^* \in D^f(n) \\ & \text{and } t=1,2,3... \end{array} \right]$$

$$S(n) = \left[s^* \in S \;\middle|\; \begin{array}{ll} \text{if } n \in W \text{ then } s^*_t = n & \text{for some } t=1,2,3... \\ \text{if } n \in D,\ n \neq n_0 \text{ then } v^*_t = n & \text{for some } t=1,2,3... \\ \text{if } n \in R, \text{ then } s^*_t \in W^f(n) & \text{for some } t=1,2,3... \\ \text{if } n = n_0 \text{ then } s^* \in S & \end{array} \right]$$

$$H(n) = [\, h \in H \mid \langle h^a_t \rangle_{t=1,2,3...} \in A(n),\ \langle h^s_t, h^v_t \rangle_{t=1,2,3...} \in S(n) \,]$$

From the branching condition defining the tree it follows that if

$n^1 \in N(n^2)$, then $A(n^1) \subseteq A(n^2)$, $S(n^1) \subseteq S(n^2)$, and $H(n^1) \subseteq H(n^2)$.

Also, $A(n_0) = A$, $S(n_0) = S$, and $H(n_0) = H$.

The whole tree structure is captured in the collection:

$$< H(n) >_{n \in N}.$$

Strategies

In the tree structure a **strategy** is a collection of actions - one action x_d for each decision node $d \in D$. The set of all initially possible strategies, denoted X, is the set of collections:

$$X = [\, x = < x_d >_{d \in D} \mid \text{ such that } x_d \in R^f(d) \text{ all } d \in D \,]$$

For each strategy $x \in X$ there are several associated mappings from S. The sequence of eventual actions a is uniquely determined for each $s^* \in S$:

$$a(s^*, n_0, x) = \left[\begin{array}{l} < a_t >_{t=1,2,3,\ldots}, \text{ such that} \\ \qquad a_1 = x_{n0} \\ \qquad a_t = x_d \text{ for } d \equiv v^*_{t-1} \text{ all } t = 2,3,4\ldots \end{array} \right]$$

The total sequence h is also determined by x for each $s^* \in S$:

$$h(s^*, n_0, x) = \left[\begin{array}{l} < h^a{}_t, h^s{}_t, h^v{}_t >_{t=1,2,3\ldots}, \\ \qquad \text{such that } < h^a{}_t >_{t=1,2,3\ldots} = a(s^*, n_0, x) \\ \qquad \text{and } < h^s{}_t, h^v{}_t >_{t=1,2,3\ldots} = s^* \end{array} \right]$$

Let $R^c(n)$ be the set of resolution nodes, and $D^c(n)$ the set of decision nodes, in the unique node sequence preceding n:

$$N^c(n) = R^c(n) \cup D^c(n).$$

A restricted strategy $x(n)$, $n \in N$ is defined as a collection:

$$x(n) = \left[\begin{array}{lll} <x_d>_{d \in D}, \text{ such that } & x_d \in R^c(n) & \text{all } d \in D^c(n) \\ \text{and} & x_d \in R^f(d) & \text{all } d \in D(n), \\ & & \text{all } d \in D^c(n). \end{array} \right]$$

The set $X(n)$ is the set of all such collections; note that some decision nodes still have their actions fixed by $x(n)$ even though knowledge that node n is visited means these other nodes cannot be visited. Also, knowledge that n is visited and $x(n)$ is followed limits the sets of feasible sequences:

$$S(n,x) = \{s \in S(n) \quad \text{for all } t = 1,2,3..., s_t \in W^f(x_d) \text{ for some } d \in D\}$$

$$A(n,x) = \{a \in A(n) \quad \text{for all } t = 1,2,3..., a_t = x_d \quad \text{for some } d \in D\}$$

$$H(n,x) = \{h \in H(n) \quad <h^a_t>_{t=1,2,3...} \in A(n,x)\}$$

4.2.3 Probabilities

The structure of the tree contains implicit likelihood statements. At the initial time the only sequences of future happenings which can be attributed positive probability are the sequences $h \in H$, and if a precommitment strategy $x \in X$ can be enforced only the sequences $h \in H(n_0,x)$ can be attributed a positive probability.

Let $$M_T(S(n,x)) = \bigcup_{s \in S(n,x)} <s_t, v_t>_{t=1,2,3...T}.$$

This is the set of sequences observable before T, when strategy x is followed and node n occurs.

Let the probability of an event $w \in W$, conditional on occurrence of node r such that $w \in W^f(r)$, be $P^R(w)$. The probability of a choice procedure resolution $d \in D$, conditional on occurrence of $w \in W$ such that $d \in D^f(w)$, is $P^W(d)$. Then:

$$0 < P^W(d) \leq 1, \quad \text{all } d \in D, \quad \text{and} \sum_{d \in D^f(w)} P^W(d) = 1, \quad \text{all } w \in W$$

also,

$$0 < P^R(w) \leq 1, \quad \text{all } w \in W, \quad \text{and} \sum_{w \in W^f(r)} P^R(w) = 1, \quad \text{all } r \in R.$$

The probability of observing the initial elements of a sequence s^*, up until time T, when strategy x is followed and node n occurs, is defined as:

$$P^S_T(s^*,n,x) = \left[\begin{array}{ll} \dfrac{\prod_{t=1,2,3...T} (P^R(s^*_t).P^W(v^*_t))}{\sum_{m \in M_T(S(n,x))} \prod_{t=1,2,3...T} (P^R(s_t/m).P^W(v_t/m))}, & \text{if } s^* \in S(n,x) \\ 0, & \text{otherwise} \end{array} \right]$$

where s_t/m and v_t/m denote the random events at t in event sequence m.

It follows that:

$$\sum_{s \in M_T(S(n,x))} P^S{}_T(s,n,x) = 1, \quad \text{for all } x \in X, \quad \text{all } n \in N, \quad \text{all } T = 1,2,3...$$

Therefore, the 'probability' of a sequence of happenings h, given that a decision x_d at each decision node $d \in D$ is enforced and node n occurs, can be represented by the sequence:

$$(P^H{}_T(h,n,x))_{T=1,2,3...} = (P^S{}_T(s,n,x))_{T=1,2,3...}$$
$$\text{where } s = <h^s{}_t, h^v{}_t>_{t=1,2,3...}$$

4.2.4 Recursive thinning

Until now, the origins of the strategies $x \in X$ have not been specified. However, in the recursive structure, at each decision node $d \in D$ the choice procedure v is fixed. By definition this identifies a single 'best' action on the basis of the actions and resolutions which are forecast to follow, and their likelihoods. This allows the tree specified so far to be reduced to 'choosable' possibilities, or thinned.

If at a decision node d^1 the 'best' action $K_d \in R^f(d)$ has been identified for each following decision node $d \in D(d^1)$ then the set of possible sequences of happenings can be identified for each action $r^1 \in R^f(d^1)$. That is, the consequences of each action r^1 are now a **probability** tree.

The collection of actions

$$y(r^1) = <r^1, <K_d>_{d \in D(d^1), d \neq d^1}>$$

contains an action for every member of $D(d^1)$. Therefore, the set of strategies $\underline{X}(y(r^1),d^1)$ employing these actions can be constructed:

$X(y(r^1),d^1) = \{\underline{x} \in X \mid \underline{x} \in X(d^1);\ x_{d1} = r^1;\ x_b = K_b \text{ for all } b \in D(d^1), b \neq d^1\}$

These strategies all limit the possible sequences after d^1 in the same way:

$$S(d^1,\underline{x}^1) = S(d^1,\underline{x}^2),$$

$$A(d^1,\underline{x}^1) = A(d^1,\underline{x}^2),$$

and $$H(d^1,\underline{x}^1) = H(d^1,\underline{x}^2), \quad \text{for all } \underline{x}^1,\underline{x}^2 \in \underline{X}(y(r^1),d^1),$$
$$\text{for all } r^1 \in R^f(d^1).$$

Therefore, the set of possible sequences of happenings after d^1, if r^1 is adopted at d^1, is the same set $H(d^1,\underline{x}')$ as might occur if precommitment strategy $\underline{x}' \in \underline{X}(y(r^1),d^1)$ was enforced. Likewise $(P^H{}_T(h,d^1,\underline{x}'))_{T=1,2,3...}$ is the 'probability', conditional on occurrence of d^1, that h will occur if r^1 is adopted. The collection:

$$< H(d^1,\underline{x}'), < (P^H{}_T(h,d^1,\underline{x}'))_{T=1,2,3...} >_{h \in H(d^1,\underline{x}')} >_{r1 \in R^f(d^1)}$$

contains the entire structure and likelihood of the outcomes h which are believed to possibly follow after each decision r^1 at decision node d^1.

Since all actions after d^1 are fixed, each action's collection amounts to a probabilistic tree over occurrences. Since each consequence q_t is a function of preceding occurrences, a probabilistic tree over consequences is also available for each action.

The choice procedure at d^1 is given. This by definition identifies, on the basis of the probabilistic trees following the actions, an action $K_{d1} \in R^f(d^1)$ which provides the 'best' such tree. Equivalently, it is initially believed that action K_{d1} is adopted if and when decision node d^1 occurs. Therefore, if $r \in R^f(d)$ but $r \neq K_{d1}$ then r will never be adopted, and no sequence of happenings in the sub-tree N(r) can occur. This sub-tree can therefore be discarded, or thinned, from the initial belief structure.

4.2.5 The initial problem

The full set of beliefs ($<N(n)>_{n\in R\cup D\cup C}$, $<P^W(d)>_{d\in D}$, $<P^R(w)>_{w\in W}$) therefore reduces, after recursive thinning, to belief that only the actions $<K_d>_{d\in D}$ will be adopted. The recursion can be extended to the initial choice by letting each possible initial choice procedure $z\in V_0$ be a node preceding a copy of the whole structure from n_0. At the **initial** time there is, for each possible initial choice procedure z:

- an optimal initial action r_z,
- a collection of 'choosable' actions $y(r_z) = <r_d>_{d\in D(D_z)=D}$,
- a set of 'choosable' strategies $\underline{x}\in\underline{X}(y(r_z),z)$,
- a set of possible sequences of happenings $H_z = H(z,\underline{x})$,
- a 'probability' of each sequence $h\in H_z$, being $(P^H{}_T(h,z,\underline{x}))_{T=1,2,3...}$

That is, the full set of possible future sequences of actions, events and choice procedures, and their likelihood, is fixed once the initial choice procedure z is known. The only 'decision' remaining is what the initial choice procedure is to be. Adoption of choice procedure z amounts to simultaneous adoption of the initial action r_z, together with the belief that future occurrences are represented by the sub-tree H_z, and its associated probabilities.

Provided thinning is possible, the recursive tree structure identifies 'best' initial actions without precommitment of future actions. These are forecast to be optimal with respect to later options and values. The initial (and later) choice procedures must determine which of a finite set of probabilistic trees is best, and in doing so may employ **any** attitudes to aspects of the trees, without causing intertemporal inconsistency.

The recursive tree structure therefore better represents the human position in time. The notion of 'optimality' employed is more sound than that of precommitment optimality. Wider choice procedures can be explored. The structure meets some of the criticisms of economic investigations of non-renewable resources which are made in Chapters Two and Three. The structure has the potential to bring about a deeper understanding of such issues.

4.3 On the existence of an optimal commitment-forecast

The recursive decision tree is neither descriptive nor prescriptive for **actual** decisions. It is intended for theoretical investigations into how actions depend on beliefs, in simplified situations. The 'solution' to an exercise employing the recursive approach is knowledge of this dependence, rather than just one optimal initial action and forecast. However, the approach requires that a thinned tree can be derived, and this may not always be possible.

Necessary and sufficient conditions for the existence of solutions to the precommitment special case have been widely investigated (Astrom, 1970; Arrow and Kurz, 1970). Few theoretical investigations derive solutions which are proven to meet sufficient conditions for precommitment optimality (Seierstad and Sydsaeter, 1983). The conditions do not apply to recursive structures. Conditions ensuring that a recursive decision tree can be thinned are discussed in this section.

4.3.1 The meaning of non-existence

Among the later elements of any partly thinned tree are forecast actions, conditional on the system reaching a future decision node. A forecast of non-existence of an appropriate action at some future node involves forecasting **no** action at that node, which is an unreasonable initial belief given that some action **must** always be taken. A forecast which involves non-existence of one appropriate action can be discarded as irrational: the reasons for non-existence may provide clues as to why the associated beliefs are unreasonable.

An optimal commitment must exist for each initial choice procedure for the same reason as with forecasts: one action **must** be taken, given that the 'do nothing' options (thinking, suicide) are defined to be actions. A choice procedure for which no optimal commitment exists is inadequate, and must be discarded. As with the forecast optima, non-existence is taken as evidence of an irrational belief rather than an intransigent problem.

Fundamentally, an optimal commitment-forecast exists if two conditions are met: firstly, the initial and all following choice procedures must be suitably discriminatory, capable of singling out one single appropriate action on the basis of the nature of the action and its ramifications; secondly, the future possibilities must allow the thinning procedure to operate.

4.3.2 Well-defined choice in recursive trees

The recursive tree structure as defined provides each choice procedure with a finite set of options. Choice procedures on such a space are satisfactory for use in recursive analysis if they always select one optimal option from any set of options.

The usual economic conception of rational choice is that choices are made in accordance with a pre-order on the options. This leaves open the possibility of multiple optima. The assumption that a strong order prevails, so each optimum is unique, seems to unnecessarily limit the range of acceptable choice procedures for use in recursive tree analysis. It seems desirable to be able to investigate **whatever** choice procedures might come to be used, irrespective of their conformity with the rationality postulates of transitivity in choice, and so on.

The standard adopted is therefore that the forecast choice procedure must be well-defined as opposed to rational, and must identify one action for selection from a finite set. A choice procedure might fail to do this either because there are too many (more than one) or too few (none) of the options left in the 'appropriate' set, after application of the procedure.

If non-existence is due to multiple optima, it must be possible to add extra discriminatory power (as when overtaking rules are used to decide between unbounded integrals), or to add a final arbitrary selection. If non-existence is due to the application of absolute standards which no option can meet, then relaxation is necessary.

Well-defined choice procedures are therefore imagined to consist of a sequence of stages of elimination of inferior options, which are carried out stage by stage. Eventually either only one option is left, or all stages have been applied or application of the next stage eliminates all remaining options. In the former case the one remaining option is selected.

In the latter cases the initial decision-maker does not know how the arbitrary future choice will be made. A forecast of uniformly distributed random 'choice' on the remaining set of options seems to best represent the possibilities. This can be incorporated in the existing recursive tree structure, by replicating the sub-tree following the choice procedure resolutions node which precedes the tied decision. Each replication contains one of the tied actions, and follows a 'perturbed' choice procedure which selects that action. Uniform probabilities are attributed to the 'perturbed' choice procedures at the preceding resolution node, to bring about the desired uniform forecast on tied choices.

4.3.3 Recursion feasibility in recursive trees

If all choice procedures are well-defined then the backwards recursion solution technique, once started, can thin the tree and hence provide an optimal initial action for each initial choice procedure. This will be true if there is a date after which all possibilities can be thinned and before which backwards recursion can thin the transient possibilities.

A finite horizon would comprise such a date. Setting a horizon T in a recursive model requires not just that the **initial** decision-maker does not care about later occurrences. Instead the initial decision-maker must believe all decision-makers before T do not care about events after T. This may be unreasonable for decision-makers at times close to T. A belief in an 'Armageddon' of some sort might supply an alternative rationale for a finite horizon T.

Full thinning is also possible if each history is eventually fully deterministic, or if no further (non-trivial) choice of action is required. In either case preceding decision-makers are presented with the required sets of probabilistic trees over which to optimize.

If along each path the choice procedure and set of feasible actions are eventually stationary, and the events have become either stationary or Markovian, then standard decision-theoretic infinite horizon methods can solve for each path's concluding optimal actions. This provides a set of probabilistic trees on which recursion can begin.

A mixture of the 'path-ending' conditions may be required to be applied. Also, it is not necessary that **all** paths be completed in this fashion before recursion can begin. If some decision-makers have limited horizons in their choice procedures over probabilistic trees, then some of their actions may well be eliminated before thinning because they are dominated by other actions. That is, the eliminated action could not produce as good a tree as some other action, whatever happens during thinning.

It is possible that there are 'infinitely transient' paths which prevent recursion from beginning, and so prevent the tree from being thinned. Forms of optimality in the limit, for these cases, are an area for future work. Further discussion of the types of paths which are sensible as rational belief structures, in an uncertain world, could also throw light on this issue.

If the world really is always transient, in some non-dominated possibilities, and in some attributes which matter enough to influence initial choice, then the only justification for any non-dominated action is that 'something must be done'. In this case a fundamentally arbitrary choice (among the non-dominated actions which cannot be 'valued') is necessary and not irrational in the usual sense of the word.

An interesting implication of irreducible uncertainty is that very specific forecasts of the very distant future are very likely to be wrong. This suggests introducing some vagueness by specifying actions, events and outcomes only up to sets, which in general grow fuzzier with time. The beliefs about the system's membership of such sets may well eventually become stationary, and fulfil the horizon setting conditions. Vagueness, therefore, may underpin rationality, in that no consistency can otherwise be guaranteed for a long-term Bayesian forecast which admits that there are future decision-makers.

4.4 Extensions

The recursive tree formulation is one of a suite of potential formulations, each of which preserves the key features of the recursive decision approach. That is, each has an initial Bayesian rational set of possible intertemporal sequences, which is successively reduced by application of a collection of decision-makers choice procedures. Eventually an optimal initial commitment-forecast is produced. This section discusses some of the difficulties to be overcome in formulating other members of the suite.

4.4.1 Discrete time recursive programs

If time is discrete, but actions, events, and choice procedures fall into continuous rather than discrete sets, then a recursive program rather than a recursive tree is required to represent a recursive set of beliefs. As before, wide sets of prior possibilities, ignoring choice consistency, contain sets of prior possibilities which are consistent with beliefs about future choice procedures. The latter sets cover the consequences of adopting an initial choice procedure.

This recursive structure employs the convention that at each time the decision precedes the resolution as to events and new choice procedure. Other tenable conventions, which also take beliefs to be recursive, treat the two sorts of resolution (as to state-of-the-world) as being jointly determined, or as determined before the decision.

The continuous action and event spaces assumed in recursive programming structures provide each choice procedure with a continuous set of commitments. The arguments of each choice procedure, however, may range over the following commitment-forecasts, or stochastic processes, governed by later chosen actions. This option set may not in general be convex, or closed.

As for the non-convex decision-tree formalism, well-defined choice procedures on such a space, for use in recursive analysis, need to ensure that they do not eliminate all options and that any final set of appropriate actions is reduced to one by a random selection. Well-defined choice procedures need not be rational in the sense of conforming to a (pre-)order on the commitment-forecasts.

In practical optimization, and perhaps in recursion in belief structures, non-convex and open option sets cause analytical problems. One is discussed further below. As for discrete space representations, a starting time is necessary to allow backwards recursion to sequentially define the optimal forecast. The same ways of generating a starting position apply.

4.4.2 Recursive and dynamic programs

The recursive programs resemble stochastic dynamic programs (see e.g. Bellman, 1957; Sengupta, 1982), multistage programs with recourse (see e.g. Dantzig, 1963; Hansotia, 1980), and stochastic control models (see e.g. Astrom, 1970; Chow, 1975). These are all models of sequential choice, for which a backwards recursion procedure reveals or locates the optimal or appropriate values of decision variables.

In general, however, the recursive program **cannot** be formulated in any of the latter three ways, because:

- in the former there is a separate objective (or choice procedure) at each stage (or time) whereas in the latter there is one overall objective which may be separated stage by stage;

- in the former, each objective in general extends over later times so that objectives ‘overlap’, whereas in the latter the value of the end-of-stage state carries all the information about the value of future possibilities;

- the former finds the current and future actions which are appropriate in the light of current and future objectives,

> whereas the latter find the precommitment strategy which is best according to current objectives alone.

As presented, the recursive program also differs from the usual form of the latter programs in that intertemporal relationships are not necessarily Markovian, and a state space has not explicitly been defined, but these are not necessary points of difference.

4.4.3 'Solution' existence and open sets

The very general recursive program structure, as specified, does not ensure that each choice procedure is provided with a closed set of options (commitment-forecasts). In particular, for some histories, the mapping from current actions to future consequences may contain jump discontinuities, even when all action spaces are closed, if there are threshold effects in **future** choices. This raises the possibility, for choice procedures of optimization type, that no 'best' action exists because the choice procedure supremum is on the (unattainable) boundary of the open set of possibilities.

Such choice procedures (utility function maximization, etc) are not sufficiently well-defined to qualify for use in (i.e. to be forecast as possibly occurring in) recursive programs where the open set problem arises. To be tenable within such a forecast these procedures must be supplemented with additional steps which determine the appropriate action whenever the open set problem arises.

One such possibility is to append a 'quantum' assumption. If there is a smallest unit by which measurements of action can differ, then continuity is a convenient approximation to the underlying discrete action space. When continuity causes problems the representation reverts to the discrete space.

For non-renewable resource actions there are often 'natural' units which provide bases for a discrete representation. Molecules provide a lower bound to unit size, measurement technology may set a much higher lower bound, and current economic practice almost certainly adopts much larger units still. Achievable transformation **rates** are not continuously variable either, given the limitations of measurement and practice.

4.5 The representation of beliefs

The recursive tree is set up as a representation of beliefs about the future, held by an hypothetical initial decision-maker. Adoption of the recursive tree structure makes explicit some beliefs which are generally not considered, but limits the beliefs which can be investigated. The relationship between the structure and possible beliefs is briefly discussed in this section.

4.5.1 Bayesian rationality

The primitive likelihood judgements in the recursive tree are the sets of P^R and P^W. The method of construction of probabilities of sequences s and h, conditional on a commitment-forecast x and occurrence of a node n, ensures that each well-defined set of probability judgements is consistent in the Bayesian sense.

That is, for any commitment-forecast, the probability, at an early node n, of any set of sequences, is the probability of the set at a later node $n' \in N(n)$, multiplied by the probability at n that the transition to n' will occur.

The tree structure therefore represents beliefs as being rational in the Bayesian sense. This is a stringent requirement for any actual person's beliefs, given that people do not consistently form probabilistic beliefs for simple gambling situations, as is outlined in Chapter Two. However, Bayesian rationality is a standard assumption in economic investigations, perhaps because alternative assumptions involving **systematic** deviation from Bayesian rationality do not yet offer equivalent analytical tractability.

The tree structure incorporates only one set of Bayesian consistent judgements for the whole set of decision-makers. That is, the initial decision-maker believes later decision-makers will adopt the **same** beliefs, appropriately conditioned to reflect the intervening occurrences. This seems to be the only internally consistent way of relating the beliefs of a sequence of Bayesian rational decision-makers. This is because if the initial decision-maker believed future beliefs might drift, then this would be incorporated in initial beliefs as a possibility, restoring the full consistency.

In reality, a fully Bayesian rational forecast must be based on some totally arbitrary judgements. For example, probabilities attributed to the outcomes of a new experiment must be arbitrary. Section 4.1 outlined how the existence of arbitrary judgements may mean that a **set** of recursive trees better represents rational beliefs.

4.5.2 Choice procedures, dynamics and uncertainty

The choice procedures may be considered to be value positions, or ethical statements about what 'the good' might be. Therefore, the (stochastic) evolution of choice procedures represented by the pathways in the recursive tree is a statement about the way values evolve; that is, about value dynamics.

Taking a particular action is compatible with only some value positions. Given that value positions do not evolve exogenously, but rather in the light of experience and ethical explorations, then only some value positions can consistently be forecast to follow from any action determined by a value position. The alternative is that the dynamics of value change are not restricted. The choice procedure can then change so 'fast' as to allow any future set of actions to be consistently forecast.

The requirement for consistency, between the 'factual evolution' and the 'value evolution' highlights the importance of the assumptions about dynamics which underpin the recursive formalism. The consistency issue is foremost as the link between current values and future possibilities is examined: is there a discontinuity between the present choice procedure (values) and future procedures, or does the future 'grow out of' the present, and if so, how quickly can it mutate to new forms?

The irreducible uncertainty about 'the good', as **initially** perceived, is partially represented. Each notion of the good is captured in one initial choice procedure which indicates one commitment-forecast is optimal. The set of initial choice procedures considered may range over the set of notions of the good which are considered justifiable to some extent.

The initial 'decision' as to choice procedure is assumed to be arbitrary - any initial choice procedure might be adopted without regard for the history. However, the set of choice procedures considered by the initial decision-maker must implicitly be conditioned by history. The set considered presumably reflects the volatility in value positions felt by the initial decision-maker, who is aware of the value positions adopted in the recent past, and is likely to have been influenced by those value positions.

This initial uncertainty about value positions is never resolved - rather, in adopting action, a 'decision' is made as to which ethical positions are most acceptable. This is akin to a definition of self on the part of the initial decision-maker.

In every recursive tree structure, irreducible uncertainties about **future** conceptions of the good are ignored; all future choice procedures are covered with a subjective probability distribution. As for general uncertainty a **set** of thinned trees may better represent a rational set of beliefs about future choice procedures.

4.6 Conclusions

The recursive decision approach appears to be a workable generalization of the decision-theoretic approach. Forecasting the choice procedures of future decision-makers enables a sequence of decision-makers to be modelled. Therefore the position in time of decision-making is better represented than in decision-theoretic approaches.

The recursive approach more correctly reflects the initial decision-maker's power to determine events. The 'implementable' initial commitments are distinguished from later recourse actions, chosen and implemented by later decision-makers. The recourse actions are, in recursive decision models, part of the forecast of the consequences of initial action.

Bayesian rationality must be attributed to the initial decision-maker, and the same beliefs must be adopted by later decision-makers, for the recursive tree structure to be fully specified. This demanding rationality postulate may be weakened by allowing that a **set** of recursive trees best represents situations where there are irreducible uncertainties.

For the recursive tree to be 'solved' or thinned it is sufficient that all choice procedures are well defined, and a starting point for backwards recursion is available or able to be constructed. Recursion can begin, for example, if there is a horizon or if choice procedures are eventually intertemporally consistent along each possible path.

The recursive decision approach can investigate very general choice procedures, because each procedure is applied sequentially. There is therefore no need for the choice procedure to be consistent with any other, for a 'solution' to be obtained. There is also no requirement that the choice procedure conform to the usual economic conception of rational choice as a (pre)order on the option space. The recursive approach seems capable of underpinning investigations of choice in dynamic, uncertain situations, where the choice procedures may both change with time and incorporate wide sets of attitudes to future outcomes.

Chapter Five

Further Features of Recursive Choice

The recursive decision structure is a conceptual framework which is potentially of use for investigations of non-renewable resources. This chapter seeks to further establish the nature and potential of recursive approaches.

Firstly, a simple two-period representation of optimal non-renewable resource is used to bring out the differences between recursive approaches and the well-known decision-theoretic special case. This is followed by a discussion of choice procedures for use in recursive structures. Reference is made here to the SEU procedure, which predominates in precommitment investigations. Analytical procedures for recursive structures are then briefly considered, before some ways of representing commitment-forecast solutions are outlined. Conclusions then follow.

5.1 A two-period representation

The differences between the precommitment decision-theoretic and commitment-forecast recursive approaches can be illustrated for a simple context by expanding on the familiar two period diagram of optimal resource depletion, as shown in Figure 5.1.

Curves A and B represent the marginal net benefit of resource use from stock K in the first and second periods respectively, as seen by the initial decision-maker. That is, the curve B is as forecast and valued at the **initial** time. In general the later net benefit is presented as a discounted monetary quantity, with the discounting representing the opportunity costs of investment (i.e. holding stocks to the second period).

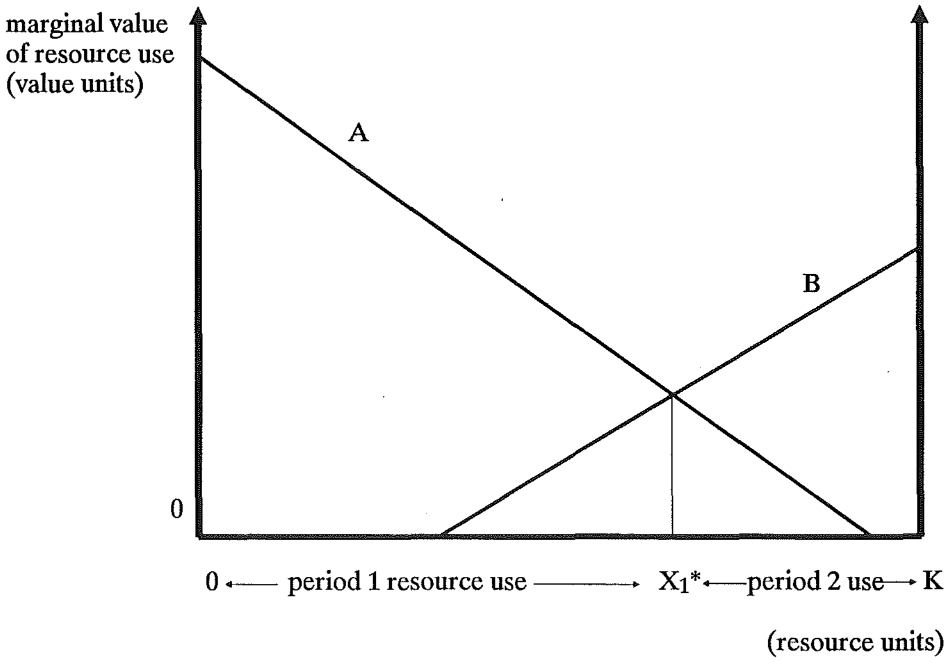


Figure 5.1: Optimal two-period resource depletion

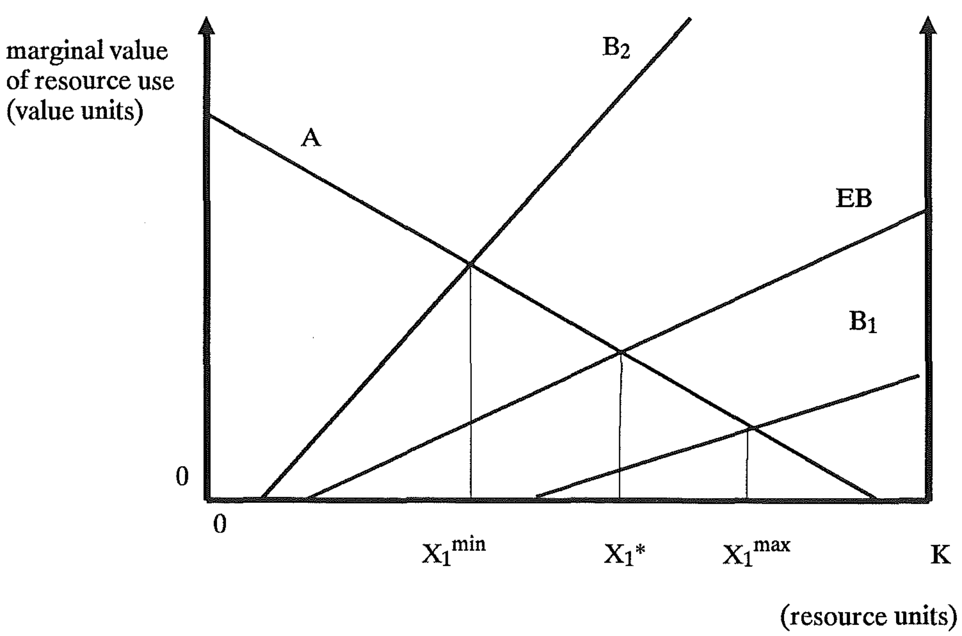


Figure 5.2: Two-period uncertainty

The optimum initial use is identified at X^*, where the marginal net benefits are equated so the sum of the benefits cannot be made any bigger by readjusting resource use patterns.

5.1.1 Uncertainty

Initial uncertainty about future events enters the diagram via the location of curve B. Assuming the monetary evaluation is adhered to and that the perceived set of possible events, and their perceived likelihood, is independent of the initial resource use, a set of locations for B is initially seen as possible, as is shown in Figure 5.2.

B_1 and B_2 are seen as possible, and given their likelihoods, the marginal expected monetary value EB of second period resource use can be calculated. The widely used risk-neutral objective employs this expected value in place of B, so the corresponding optimum is at X_1^*. This initial choice procedure is event separable, so changing the likelihoods shifts EB and hence X_1^*, which can therefore be derived as a function of the likelihoods.

Figure 5.3 illustrates how the optimum varies with varying belief. Let p_1 be the likelihood initially attributed to the outcome giving B_1 in Figure 5.2, and since by construction there are only two possibilities, $1-p_1$ is the likelihood attributed to the outcome giving B_2. For each belief, that is each p_1, the risk-neutral optimum X_1^* can be calculated, and will be somewhere between X_1^{min} (for $p_1 = 0$) and X_1^{max} (for $p_1 = 1$).

The belief p_1 is a point on a one-dimensional simplex in Figure 5.3. If there were $n > 2$ discrete possibilities here the belief-optimum mapping would be a surface over an n-1 dimensional simplex. There is one surface (choice function) for each well-defined choice procedure.

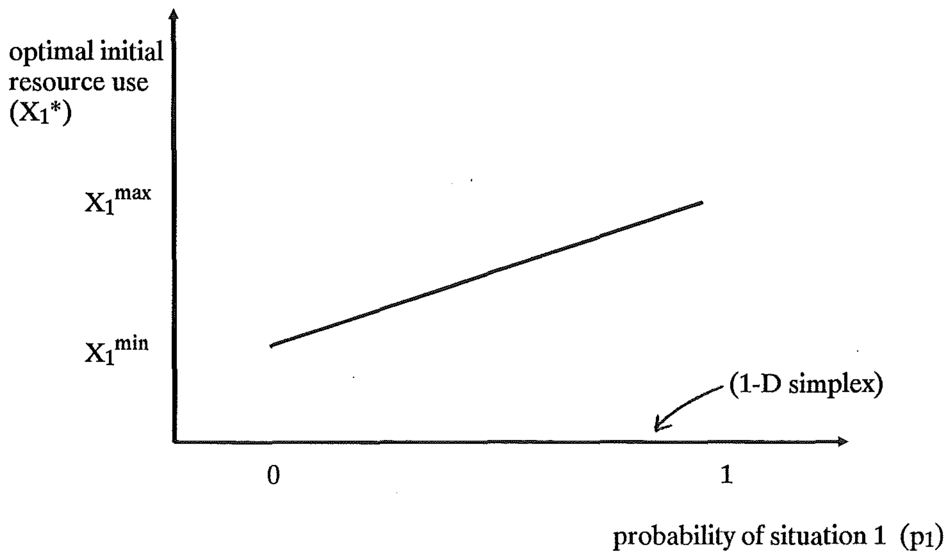


Figure 5.3: Optimum as a function of forecast beliefs

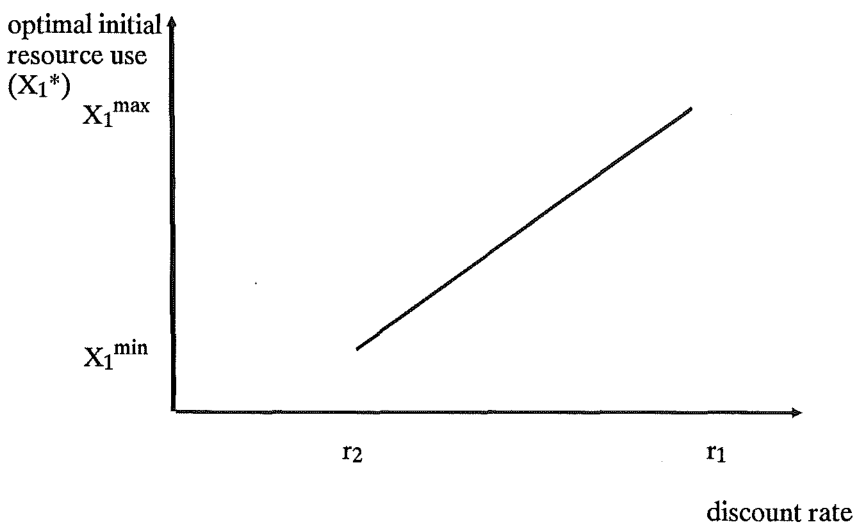


Figure 5.4: Optimum as a function of initial choice belief

5.1.2 Choice procedures

Initial uncertainty about which choice procedure to apply enters the original certain diagram (Figure 5.1) via the location and relevance of curves A and B. For instance, there is a class of choice procedures which all involve adopting that action which maximises a discounted monetary sum, but which vary in their discount rate. Assuming that the initial conditional forecast (of second period resource use and its consequences) is invariant to the initial choice procedure adopted, the discount rate possibilities give a set of positions for B and hence a set of X_1*. Figure 5.2 could represent two choice procedures, instead of two outcomes; B_1 is the initial valuation (of the second period choices) using a higher discount rate procedure and B_2 is the initial valuation for a lower discount rate procedure.

If the choice procedure differences can be captured with a parameter (here the discount rate), then the initial optimum can be written as a function of that choice parameter. In Figure 5.4 the discount rate varies between a maximum of r_1 and minimum of r_2, corresponding to the valuation curves of B_1 and B_2 in Figure 5.2 respectively, and hence the initial optimum resource use of X_1^{min} and X_1^{max} respectively.

The recursive approach differs from the decision-theoretic approach in that the second period resource use is forecast and then valued by the initial decision-maker. That is, second period resource use is not initially chosen, although the second period possibilities may be influenced by initial actions.

The curves B_1 and B_2 in Figure 5.2 could represent initial uncertainty about how the second period decision-makers will choose to use the remaining resource. The initial decision-maker attributes the higher value B_1 to one sort of future use, and the lower value B_2 to a different future use, but cannot dictate the future choice from the initial time period.

5.2 Choice procedures for recursive decision models

This section briefly discusses the nature of the choice-procedures which are consistent with making decisions **in** time, recognizing that there will be a sequence of decision-makers. The decisions which arise in any recursive structure are decisions between commitment-forecasts, such as probabilistic trees.

5.2.1 Justifiable choice procedures

To be justifiable for use in a recursive structure, a choice procedure for deciding between commitment-forecasts must satisfy two 'rationality' requirements; beyond this arguments about scientific standards, and about ethical standards, apply. The first rationality requirement is that the objects of choice - those things which enter into the choice procedure - must be drawn from a consistent recursive tree (or equivalent continuous structure). This tree captures the beliefs about the consequences of possible commitments, in a form which has eliminated logical inconsistencies to the fullest extent possible. The tree prevents overstatement or understatement of each decision-maker's power to commit the system to action.

The tree also eliminates from consideration 'possibilities' which violate the 'laws' the decision-maker accepts as governing the system's behaviour. These laws are likely to be either generally accepted claims, such as that a thing cannot be in two places at the same time, or currently accepted 'scientific' knowledge, such as the thermodynamic laws. Because there is no way of proving scientific knowledge to be correct, the justifiability of any internally consistent tree still remains a matter for argument.

The second rationality requirement is that application of the choice procedure always results in identification of a determinate (perhaps singular) set of appropriate - or adoptable - commitments, from which the adopted action may be chosen using a random procedure with known parameters. This requirement ensures that the choice procedure has sufficient content to be defined and hence discussed. Such choice procedures as 'picking the commitment which is the third one to occur to me' are ruled out. The requirement makes rational choice a **deliberate** undertaking.

The ethical justifiability of a choice procedure is an additional issue. Every choice procedure has an implicit moral community, and may implicitly extend rights of various sorts, take various interests into account, and arbitrate conflicting rights and interests in various ways. The ethical consistency of the procedure (are like interests treated alike?) may sharpen argument on its justifiability, though not conclusively, it seems.

5.2.2 Orders and utility functions

It is standard in economics to assume that any 'rational' choice procedure corresponds to a complete pre-order on the choice space (Debreu, 1959). If this is applied here, a binary relation: 'sub-tree A is at least as appropriate as sub-tree B', written $A \succsim B$, could be defined, which would satisfy:

$A \succsim A$, for all possible sub-trees A,

and if

$A \succsim B$ and $B \succsim C$ then $A \succsim C$, for all possible sub-trees A, B, and C,

and either

$A \succsim B$ or $B \succsim A$ (or both), for all possible sub-trees A and B.

If choice conforms to a pre-order there may be classes of sub-trees which are of indistinguishable appropriateness, including the 'most appropriate', or maximal set. To 'rationally' identify a **uniquely** appropriate action the choice procedure must be more discriminating, and correspond to a strong order. This defines a binary relation which satisfies the above properties, and also:

if $A \gtrsim B$ and $A \lesssim B$ then $A = B$, for all possible sub-trees A and B.

The assumption of random choice from the maximal set seems more justifiable than assuming a strong order; the exact underpinnings of choice are not critical to the recursive **structure** as a rational model of the **setting** for choice.

It is well-known that a strong order cannot in general be represented by a utility function, and that a complete pre-order cannot always be represented by a utility function. The finite option sets of recursive tree models allow pre-orders to be represented by utility functions. However, choice procedures as heuristically outlined are not representable as a matter of course.

It is usual in economics to assume that the arguments of preferences or utility are the outcomes q_t (and their probabilities) only. Further arguments for choice procedures are discussed below.

5.2.3 Expected summed discounted utility

In standard precommitment formulations the decision-maker's choice procedure is to maximise the expected intertemporal integral of discounted instantaneous utilities. These utility 'rates' are, in non-renewable resource use contexts, generally a direct function of the current resource use rate only. The utility 'rate' function is concave in the resource use rate, and so therefore is the integral and its expectation: the decision-maker is risk-averse towards each future time's resource use.

As between programs of utility delivery, however, the decision-maker is risk-neutral, as is outlined in section 2.3.4.

In applying this choice procedure to commitment-forecast options for non-renewable resource use, the resource output of each time/state is valued and summed for each branch in the tree. The event probabilities are used only to form a branch likelihood measure, for calculating the overall expected value of the tree. The optimal commitment is the action which maximises this measure.

The discounting of future utility which is central to this choice procedure is an ethical position which is subject to argument when certainty prevails, but these arguments are beyond the scope of this work. Discounting when there is uncertainty introduces another ethical judgement: the later discounted utility 'rate' functions are less concave than the early functions, so the decision-maker is less averse to later utility risks than to earlier ones, and may judge large later risks as less worrying than smaller earlier risks.

The choice procedure is a special case of the subjective expected utilitarian (SEU) procedures. Effectively, the SEU rationality axioms (which are powerful for the one-shot decision-event context) are applied here to **vector** consequences. That is, each whole branch is taken to be one possible consequence of action, rather than each sub-tree being a consequence. The choice procedure is separable not just in sub-trees (initial events), as the axioms require, but in whole branches (states-of-the-world), and in time.

Use of expected value in the choice procedure is an additional ethical judgement. Given that only one eventuality can result, is it appropriate to attempt to make this eventuality a good one 'on average', at the expense of possibly making it a bad one? In this context there is no compensation between people or between periods which can average things out in actuality. The central ethical question is whether certain actions are justifiable, in the face of the possible outcomes and the outcomes possible from alternatives. It seems ethically easier to justify actions (which might turn out to lead to poor outcomes) when they are taken to prevent **worse** outcomes. That is, a risk-averse stance seems more easily ethically justifiable.

5.2.4 Subjective expected utilitarian choice

This choice procedure is reviewed in Chapter Two. The maximization of the expected intertemporal integral of discounted utilities, discussed above, is a special case of SEU choice. There the SEU axioms are applied to vector consequences and an intertemporally separable utility function is applied. Each of these assumptions can be relaxed while maintaining SEU rationality.

First, an SEU procedure for valuing commitment-forecasts could discard intertemporal separability. The utility measure might then be formed by multiplying together a time series of attribute levels, or by selecting the minimum level over time of an attribute, for each branch. The full SEU measure for the tree would be formed as the expected value, over branches, of the utility measure. Choice with such a procedure, over probabilistic trees within a **recursive** structure, will not lead to intertemporally inconsistent choice.

Further, the state-of-the-world separability assumption could be weakened to separability in initial events only. A corresponding SEU-rational choice procedure employs a utility function on the whole sub-tree following each **initial** resolution of uncertainty. A utility function which multiplies together the attribute levels of all branches, at each time **after** the initial resolution, meets the SEU axioms. The full SEU measure for the tree would be the expected value, over possible initial resolutions, of the utility measure.

5.2.5 Attitudes to uncertainty

There is a suite of as yet unexplored choice procedures which represent attitudes to program risk, as opposed to risk at a point in time. The simplest example involves maximising the expected value of a concave function of the value of the possible programs, with each program possibility (branch) taken as a whole. This is discussed in section 2.3.4.

More extreme aversion to program risk would be reflected in a 'maximin' choice procedure: adopt that action which has the best valued program under its worst possible outcome. A special case of this is maximin over possible situations. More generally, a choice procedure representing program risk attitudes might be averse towards risk on sequences of attributes at some dates and approving of risk on sequences over other dates.

A further and different reaction to uncertainty, which can be captured in more general choice procedures, is 'aversion to (or love of) risky situations'. Here the decision maker has a desire to create or avoid more certainty, above and beyond his or her feelings about the possible outcomes themselves. It is possible that the objects of choice here are the probability judgements themselves, or some measure of them such as their entropy, but this does not seem likely - surely the **range** of some underlying variable is required in estimating what levels of certainty are desirable. More reasonably, the decision-maker is likely to have attitudes to learning and/or surprise which result in a non-neutral attitude to the temporal resolution of uncertainty. That is, some 'certain' utility will often be traded off for a change in the resolution pattern toward 'knowing sooner', or 'not finding out till later'.

Choice procedures which factor-in attitudes to program risk or to risky situations may be compatible with SEU choice, because the SEU utility arguments need not be branches. That is, the choice procedure may conform to the SEU axioms provided it is separable in the **initial** uncertainty resolution.

5.2.6 The arguments of choice procedures

The choice procedure arguments used in economic investigations are in general physical quantities, and resource throughput rates are the most common in the current context. When the human position in time is made explicit, wider features present themselves as choice arguments.

Ensuring the existence of some future persons may be paramount in some ethical systems and hence in some choice procedures. In resource context this has been approached obliquely by examining optimal resource use patterns when minimum consumption levels are required for survival.

Other ethical systems may be concerned with the **nature** of future persons. A corresponding choice procedure could employ **future** choice procedures as its arguments, since these presumably capture the value positions adopted by future persons.

The experiences of future persons are possible arguments for initial choice procedures: these experiences are captured in the sequences of whole sub-trees which represent future persons' perceptions of unfolding actions, events and outcomes. More restrictively, future wellbeing could enter initial choice procedures subject to a 'non-paternal' evaluation of future positions. This would involve use of each future's **own** valuation measure (if such is believed to exist) applied to its own situation, relative to others. That is, the initial choice procedure would not assume to know 'what's good for future people' above and beyond weighting the expected evaluations made by future people.

The state of the non-human world in later times supplies an additional plethora of arguments for consistent choice procedures which reflect the initial decision-maker's values. Positions such as minimising human impacts, or maximising species diversity, might be found here.

5.3 Analytical procedures

The recursive decision structure is a conceptual framework which helps decision-makers become informed - it gives added precision to beliefs about the justifiability of possible actions. Knowledge of the structure helps clarify what meaning observations and investigations have. More actively, the structure can be the basis for new investigations. Its active usefulness depends on how easily applications of the structure can be analysed, and this is now briefly discussed.

5.3.1 Analytical tools and their limits

In actively using the recursive structure to give some precise content to beliefs about the justifiability of actions the general methodology of normative economic theorising is applied. Beliefs (about how the world operates) are formulated in set theoretic terms or as differential equations, with stochastic elements to express perceived uncertainty. Mathematical logic enforces internal consistency, and some of the perceived indeterminacy of the future is dispelled, by identifying how choices will or should be made.

In general, this choice identification employs the necessary and sufficient conditions for some form of constrained optimization. The theoretical investigations based on precommitment formulations are generally not specific about the equations used, beyond general assumptions on continuity and differentiability, and precise numerical specification is very rare.

Analytical or closed form solutions, in which the optimal choice is an exact function of the parameters of the formulation, are generally sought. Analytical solutions which can be readily interpreted can be derived only for the simplest models. If more than three or four non-linear equations characterize the solution then the implied dependencies remain obscure.

5.3.2 Approximation procedures

There are several ways of achieving a partial understanding of a complex situation. Each way approximates the complex situation differently. Clearly, one whole feature of the situation can be ignored, or held constant, so as to reduce the number of equations to be solved. The remaining features can then be examined, perhaps for each of several different assumptions about the fixed feature.

A different approach is to make assumptions about the formulation which allow the complex situation to be treated for a special case. Assumptions of this sort which are widely used are that functions and their derivatives are continuous, or are linear or logarithmic.

Another option, not widely used, is to use numerical analysis and mathematical programming routines to explore special cases. If each case can be quickly solved an approximation to the analytical solution may be able to be built up in piecewise fashion. This method allows formulations which contain or give rise to non-convex sets of possibilities to be treated, although perhaps at the cost of a coarse approximation.

Which approximation is best depends on the purpose of the analysis. In general, an approximate solution to the right problem is more trustworthy than the exact solution to an approximate problem. If the aim in theorising is to help decision-makers by providing them with better principles or by informing them, then the exact solutions for special cases must be checked for wider relevance.

Solution relevance might be checked by comparing the exact solutions with approximate solutions to situations capturing more complexity. Discrete numerical approximations offer the possibility of doing this. Systematic techniques for approximating high-dimensional problems are currently under investigation (Geoffrion, 1977; Zipkin, 1980; Griffin, 1982).

5.3.3 Standard problems

A suite of standard problems could be established to provide some focus for the differing ways of approximating complex situations. These could be addressed in each fashion by each method in order to isolate points of difference and agreement. To an extent this would only formalise existing practice, but the formalisation seems to offer a way of keeping the **purpose** of theoretical analysis (informing decision makers), paramount. Dealing with a suite of standard non-renewable resource problems is a research program involving elements of (at least) economics and moral philosophy.

For each standard problem the different approximations produce different pieces of the belief-action mapping, and/or different suggestions about the nature of each piece. Comparisons, of the mapping implied by ignoring a factor with the mapping implied by coarse approximation of that factor, should identify those beliefs where the two mappings differ greatly. These differences help in judging the reliability of theoretical conclusions, and might indicate how future theorising should be directed so as to best guide actions; i.e. how complexity should be approached, if the goal is to produce a widely accurate belief-action mapping.

The disagreements about solution mappings are a way of obtaining theoretical advance in a situation where notional falsifiability is not workable as a criterion because the theory is inherently normative. The comparisons provide a way of eliminating theory which provides **false** precision, as against a way of giving precision (of unknown trustworthiness) to a new area, or by a new analytical routine.

In non-renewable resource context there are several 'natural' dimensions on which standard problems could differ. Open and closed economies, with and without price-setting power, with and without backstop technology available, are some of the important cases. Other 'technological' assumptions might cover whether decision-makers are becoming more or less concerned for the future, and whether they are drawing more or less relative satisfaction (choosing as if...) from consumptive or environmental experiences.

5.4 Optimal commitment-forecasts

In dynamic, uncertain situations the optimal initial actions are always the initial action of one commitment-forecast drawn from a thinned tree.

It is useful to think of a mapping, from the initial belief to the optimal initial commitment as being 'the solution' sought. The structure of this mapping clarifies the content and meaning of solutions to fully specified formulations. The mapping also helps clarify the concepts of informed decision-making, and 'justifiable' action.

Initial beliefs have two components: beliefs about future occurrences, and beliefs about 'the good', or the justifiable choice procedures. Representations of these components, and of initial action possibilities, brings out the form of the mapping.

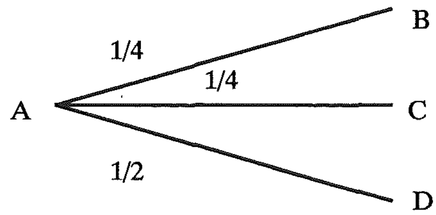
5.4.1 Representation of predictive beliefs

In a recursive tree, all initial beliefs about future occurrences are captured in the thinned tree structure, and the conditional probabilities over events. When there is a finite number K of event possibilities, the conditional probability judgement can be represented as a point on a (K-1) dimensional probability simplex as Figure 5.5 shows. Irreducible uncertainty about a probability judgement can be represented as a set of points on the simplex as in Figure 5.6. If the irreducible uncertainty extends to the set of events (node structure), then K can be redefined as the number of events in the union of the possible events given possible structures, and the K-1 dimensional simplex representation of each resolution node still applies.

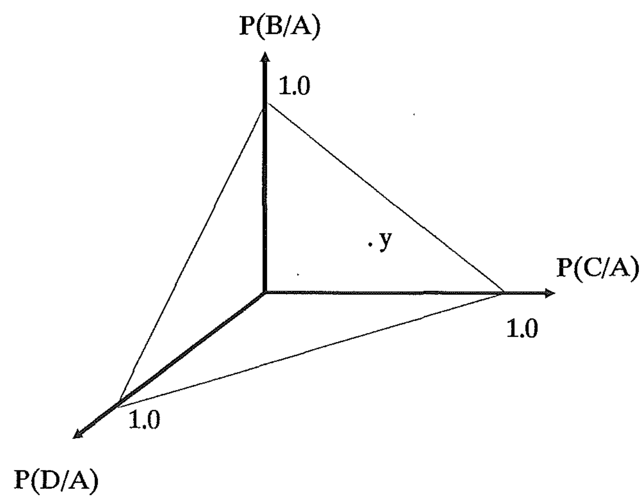
If a set L of resolution nodes (indexed i) are initially thought possible (before fixing forecast or commitment actions) then the full set of beliefs can be represented as a corresponding set of points on probability simplexes, each of dimension K_i-1, $i \in L$. If L is finite then an equivalent representation is as a point in the unit cube of dimension:

$$\sum_{i \in L} (K_1 - 1).$$

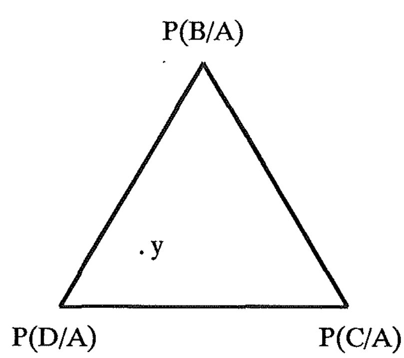
In either space a set of points or region can represent the compound irreducible uncertainty. The simplex in Figure 5.6 is maintained here as a stylistic representation of an initial belief that *a priori* reasonable internally consistent sets of beliefs about future occurrences must fall within the space B_1.



(a) in tree form



(b) in probability space



(c) on a simplex

Figure 5.5: Conditional probabilities

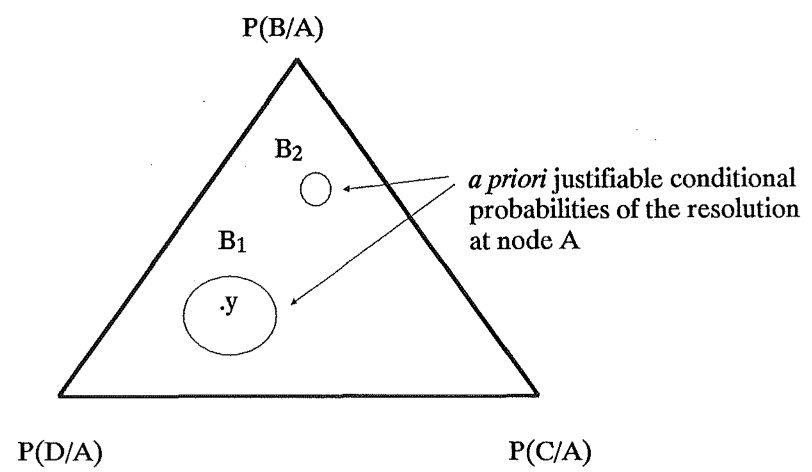


Figure 5.6: Irreducible uncertainty about conditional probabilities

Note that, for finite trees, the set of simplexes is constructed from the conditional resolution probabilities and so is collectively of higher dimension than the single simplex required to represent the probabilistic beliefs as to the eventual result, that is the branch probabilities. The set captures the temporal resolution of uncertainty, whereas the single simplex does not - it must be supplemented with a sequence of partitions of the set of branches before it contains the information required to build the complete tree.

For infinite trees, an infinite sequence of sets of simplexes is required to represent the temporal resolution of uncertainty; one set for each time period. This set may represent probabilistic beliefs as to the conditional resolutions in that period, or as to the likelihood of observable branches up to that period - each series is obtainable from the other given that Bayesian rationality applies throughout the belief structure.

Initial beliefs which vary in their probability judgements correspond to different sets in the probability space (B_1, cf. B_2). Varying tree structures can be represented by allowing that some beliefs attribute zero probability to some events, and expanding the dimensions of the probability space to account for any event which is ever given positive probability.

5.4.2 Representation of choice procedures

Ethical beliefs, or preferences, are of interest here because they influence decisions, and therefore to the extent that they are applicable through initial choice procedures. If a finite number of initial choice procedures are under consideration, their range can be represented by a set of indices. More generally, the set of procedures will vary over a number of parameters. The range of procedures can then be represented by the set V of possible parameter options.

For example, the discount rate, a non-negative real number, is a parameter for the standard precommitment procedures which maximize the integral of discounted instantaneous utilities.

If each $v \in V$ maximises a measurable 'utility' function over commitment-forecasts, then the recursive structure provides a utility value for each initial action. This provides a basis for sensitivity analysis: the scores of each action for each notion of utility $v \in V$ may indicate that some actions are dominated (another action always scores better) and can be discarded; the **measurable** utility provides an estimate of the 'distance' from optimality for each action combination.

5.4.3 Representation of the 'solution'

In a recursive tree the set of possible initial actions is finite, and may be represented by the set of indices. In other recursive structures with continuous ranges of possible initial actions the introduction of parameters may allow the initial actions to be represented by a closed set, as for choice procedures. In either case the initial actions are members of a defined set A_1.

An internally consistent belief set $b \in B_1$ and an initial choice procedure $v \in V$ are sufficient to determine the optimum initial action $a^* \in A_1$ and the associated commitment-forecast. Varying b and v over their respective ranges produces an initial optimum a* for each belief $(b,v) \in B_1 \times V$. Collecting the optima provides the 'solution' mapping: $f: B_1 \times V \rightarrow A_1$. The form of f, as b_1 and v vary, indicates which aspects of belief matter to the initial decision.

For each fixed choice procedure v, the 'solution' mapping f captures how the optimal initial actions change with b, the probabilities attributed to possibilities.

For each fixed set of beliefs about consequences b, and assuming the existence of optimal initial actions a* in all cases, the 'solution' mapping links optimal initial actions to choice procedures, as Figure 5.7 illustrates.

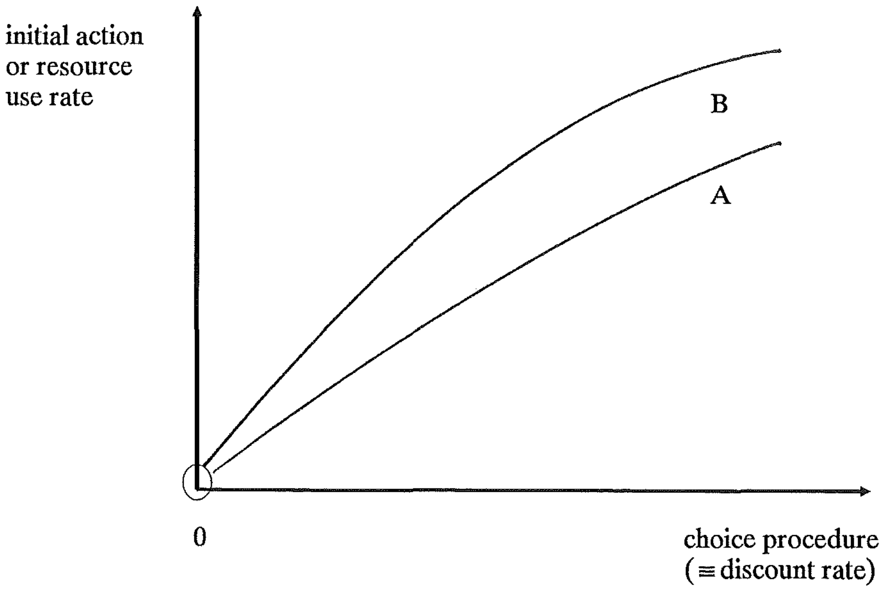


Figure 5.7: Choice procedures and optimal initial actions

In this figure the initial actions are optimal precommitment non-renewable resource use rates for 'cake-eating' beliefs about the world. Curve A is derived for fixed beliefs that the stock (cake) is of a certain size, and curve B is derived for fixed beliefs that the stock is bigger. The choice procedures vary over discount rates: no utilitarian precommitment optimum exists for a zero discount rate, and the optimum initial use increases rapidly as the future is increasingly discounted in the beliefs adopted.

When the set A_1 of possible commitments is discrete, the joint belief space $B_1 \times V$ can be partitioned according to the commitment $a \in A_1$ which is optimal for each belief $(b,v) \in B_1 \times V$. Adopting a commitment, that is undertaking an initial action, is equivalent to adopting a belief in the region(s) identified with that commitment by the partition. The more contiguous and larger are the partitioned areas of $B_1 \times V$, the easier it should be to identify appropriate actions in the face of uncertainty. Only one of the partitioned regions, not a single point, need be selected. If the set of commitments is continuous then the belief space may be contoured according to the prevailing optimal commitment, rather than partitioned.

5.5 Conclusions

The recursive decision approach focuses on initial choice, taking future choices as among the 'facts' which determine and constrain the consequences of each option. This is apparent in reconsidering the nature of the future value curves used in a standard two-period model. The focus is further emphasized by the derived 'solution' mapping, which relates **initial** beliefs to **initial** optimal actions.

A wide range of previously ignored choice procedures can be explored with recursive structures. Many of these can be found by relaxing the assumptions constraining choice procedures for precommitment formulations. The assumptions are not necessary to guarantee intertemporally consistent plans or forecasts, if a recursive approach is employed.

Routines for analysing recursive structures can be devised. If non-convex sets of consequences arise in formulations employing continuous variables then numerical approximations can provide a solution procedure.

Chapter Six

Changing Choice Procedures

Principles guiding non-renewable resource use should allow for the fact that future non-renewable resource use decisions will be taken by future decision-makers, who are likely to have objectives which differ from current ones. This is because people's tastes, and the ethical positions they hold, change over time. Use of a non-renewable resource can extend over many generations, so a considerable change in objectives may occur.

Changing objectives cannot be investigated with the precommitment formulations used to date in prescriptive economic investigations of non-renewable resource use. These formulations are in general internally inconsistent when applied to changing objectives. It is not known whether the precommitment solutions are close to solutions which allow for changing objectives. The recursive approach developed above is intended to incorporate changing objectives, without becoming inconsistent.

In this chapter the recursive approach is applied to non-renewable resource use decisions when it is known that objectives will change. The application is made for two reasons. Firstly, it is an initial test of whether the recursive approach is workable, and is capable of adding to the theoretical results which underpin resource use principles. Secondly, the application seeks an indication of the importance of the fact that objectives change. If the results of the recursive approach do not differ significantly from the precommitment results then the latter formulations may safely ignore changing objectives.

A very simple extension of the well-known 'cake-eating' model is investigated. A three-period model is formulated, with changing objectives incorporated by giving each period's decision-maker a different discount rate, within the usual utilitarian objective. The finite horizon allows the recursive formulation to be solved by backwards recursion. The optimal initial commitment is derived as a function of the discount rates for a simple utility specification. This allows a comparison with the precommitment 'cake-eating' results to be made. The chapter concludes by discussing the significance of the findings.

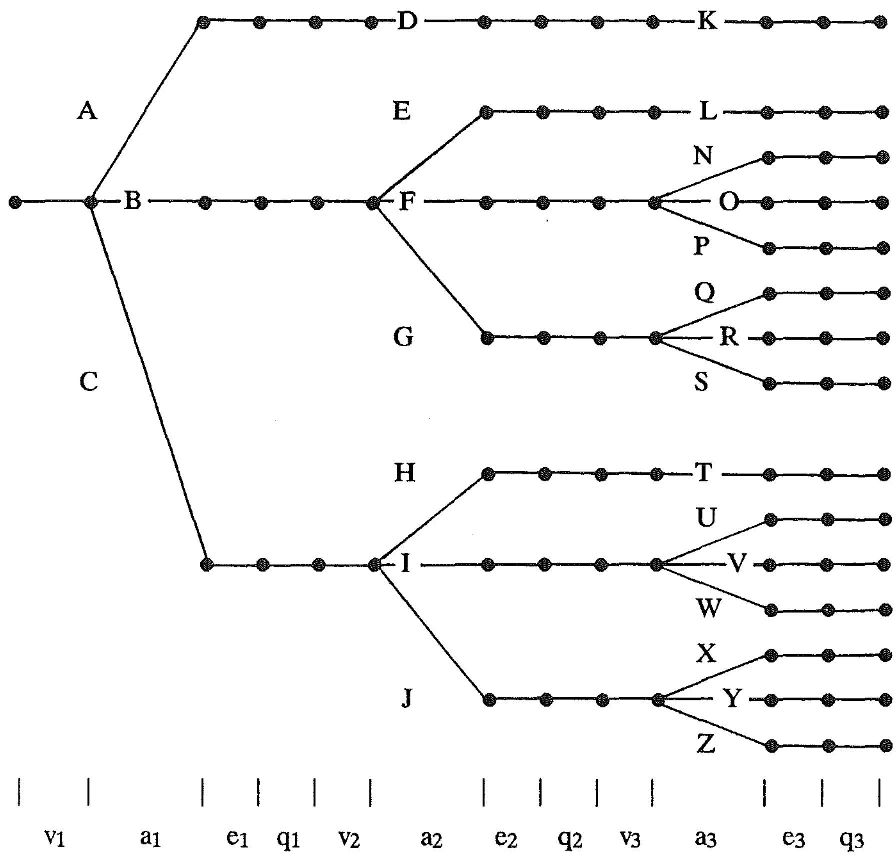
6.1 Initial belief formulation

An initial decision-maker believes that there will be a new decision-maker in each future period, and that the future extends for three more periods, indexed $t = 1,2,3$. It is further initially believed that the decision-makers in the second and third periods believe in the same horizon, so only three periods are under examination. There is no uncertainty in the beliefs.

In each period the only action is the costless throughput (extraction, use and discard) of a_t units drawn from a stock of homogeneous units of a non-renewable resource. It is initially believed that the total resource use over the three periods must be no more than M units, and that later decision-makers think this too. The choice structure is illustrated in Figure 6.1.

It is believed that future decision-makers will, like the first, adopt a discounted utilitarian choice procedure v_t, each of which takes each period's utility to be a set function U(.) of resource use in the period.

It is believed that the choice procedures v_t change over time because each decision-maker adopts a different discount rate r_t, as well as because the time remaining until the horizon differs.



Key to labels, all in resource units used:

M = maximum stock available.

$A = M \Rightarrow D = K = 0.0$

$B + E = M \Rightarrow L = 0.0$

$C + H = M \Rightarrow T = 0.0$

$C = G = J = P = S = W = Z = 0.0$; lower limits

$A + D + K = M$

$B + E + L = M$

$B + P + N = M$

$B + G + Q = M$

$C + H + T = M$

$C + J + X = M$

B,F,I,O,R,V, and Y are representative of the infinite set of branches from these nodes.

Figure 6.1: Stylistic representation of three-period resource use choice

The initial beliefs correspond to an unthinned recursive formulation:

$$v_3: \quad \max_{a_3} \quad U(a_3)$$

subject to: $a_1 + a_2 + a_3 \leq M$
a_1, a_2, M given
$a_3 \geq 0.$

$$v_2: \quad \max_{a_2} \quad U(a_2) + \frac{1}{1+r_2} U(a_3)$$

subject to: $a_1 + a_2 + a_3 \leq M$
a_3 solves v_3 given a_1, a_2, M
$a_1, M,$ given
$a_2 \geq 0.$

$$v_1: \quad \max_{a_1} \quad U(a_1) + \frac{1}{1+r_1} U(a_2) + \frac{1}{(1+r_1)^2} U(a_3)$$

subject to: $a_1 + a_2 + a_3 \leq M$
a_3 solves v_3 given a_1, a_2, M
a_2 solves v_2 given a_1, M
M given
$a_1 \geq 0$

6.2 Derivation of optimal initial action

The recursive structure is thinned to find the optimal initial action by defining a state variable R_t as the maximum resource use believed possible from period t onwards. The period 3 and period 2 optimal commitments can then be derived as functions of the state variable, which is then fixed by the first period optimum.

To simplify the solution procedure, and avoid corner solutions, assumptions on the utility function and discount rates are useful:

Let $U(.): R_{+} \rightarrow R$ be everywhere continuous and twice differentiable, with

$$U'(.) > 0, \quad U''(.) < 0, \quad \lim_{a \to 0} U'(a) = \infty, \quad \lim_{a \to \infty} U'(a) = 0,$$

and let $0 \leq r_1 < \infty$ and $0 \leq r_2 < \infty$.

The period 3 optimum a_3^*, as a function f_3 of the state R_3, is found by inspection:

$$a_3^* = f_3(R_3) = R_3, \quad \text{all } 0 < R_3 \leq M.$$

Equivalently, application of v_3, to the branches feasible at each period 3 choice node, leads to thinning of the tree of possibilities. Only possibilities where all resource available in period 3 is used in that period remain after thinning. In Figure 6.1 branches O,P,R,S,V,W,Y and Z are eliminated at this stage.

Correspondingly, the initial decision-maker believes that the period 2 decision-maker will believe that all inherited resource R_2 which is not committed to second period use a_2 will be used in the third period. The second period's choice procedure becomes:

$$v_2: \quad \underset{a_2}{\text{Max}} \quad U(a_2) + \frac{1}{1+r_2} U(R_2 - a_2)$$

$$\text{subject to:} \quad R_2 \text{ given}$$
$$a_2 \geq 0.$$

The conditions on U ensure that the period 2 optimum satisfies $0 < a_2^* < R_2$ if $R_2 > 0$, so at each v_2 optimum it is true that:

$$U'(a_2) - \frac{1}{1+r_2} U'(R_2 - a_2) = 0, \quad \text{for all } 0 < R_2 \leq M.$$

Define the functions on the positive real line:

$K_1 = (U')^{-1}$	$:R_+ \to R_+,$	inverse marginal utility
$K_2 = \left(\frac{1}{1+r_2} U' \right)^{-1}$	$:R_+ \to R_+,$	inverse discounted marginal utility
$K_3 = (K_1 + K_2)$	$:R_+ \to R_+,$	addition of the inverses
$K_4 = (K_3)^{-1}$	$:R_+ \to R_+,$	inverse of the addition; i.e. the marginal worth of R_2 to period 2.

The existence of these functions is ensured by the conditions on U. Then, the period 2 optimal commitment a_2^* satisfies:

$$U'(a_2^*) = \frac{1}{1+r_2} U'(R_2 - a_2^*) = K_4(R_2)$$

Therefore:

$$a_2^* = K_1(K_4(R_2)) = f_2(R_2), \quad \text{for all } 0 < R_2 \leq M.$$

The derivation of the functions and the optimal commitment is shown in Figure 6.2. Here K_4 is the horizontal addition of the second period's marginal valuation of period 2 and period 3 resource use. For any R_2, the optimal a_2^* and a_3^* can be derived by implicitly using the optimality conditions: the a_2^*/a_3^* split must use all R_2, and must be such that each period's use is, at the margin, contributing the same amount to the second period's objective.

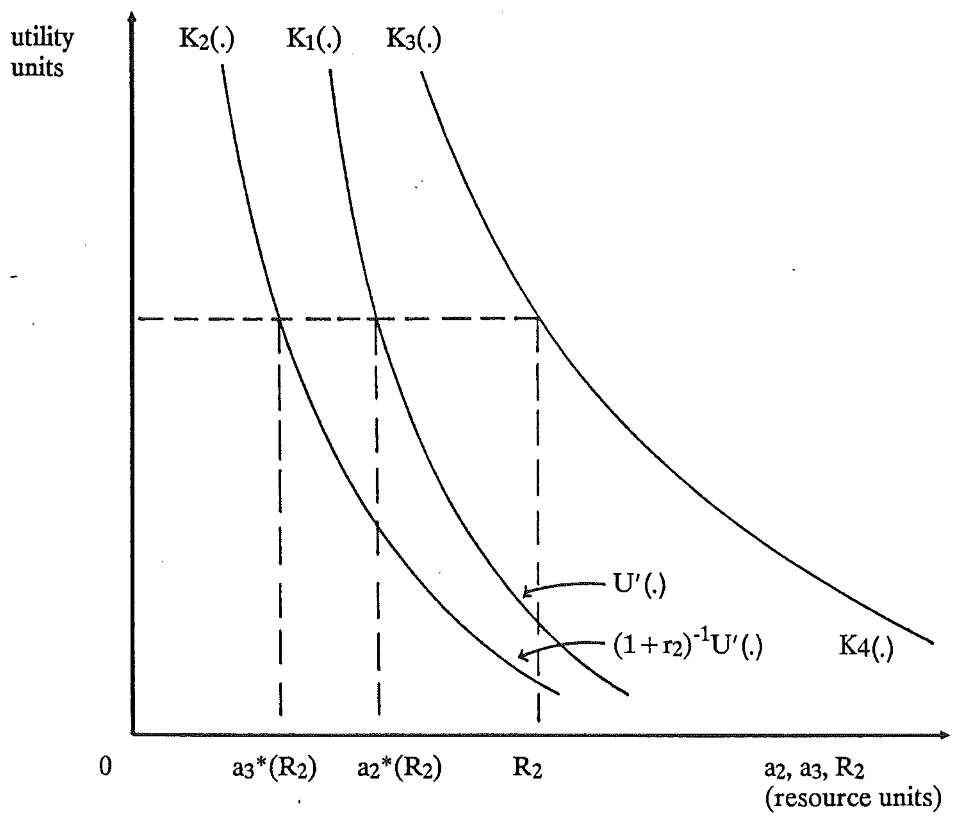


Figure 6.2: Optimal second period commitment

Following this procedure for each $0 < R_2 \leq M$, as in Figure 6.3, gives the optimal commitment as a function of the resource availability:

$$a_2^* = K_1(K_4(R_2)) = f_2(R_2).$$

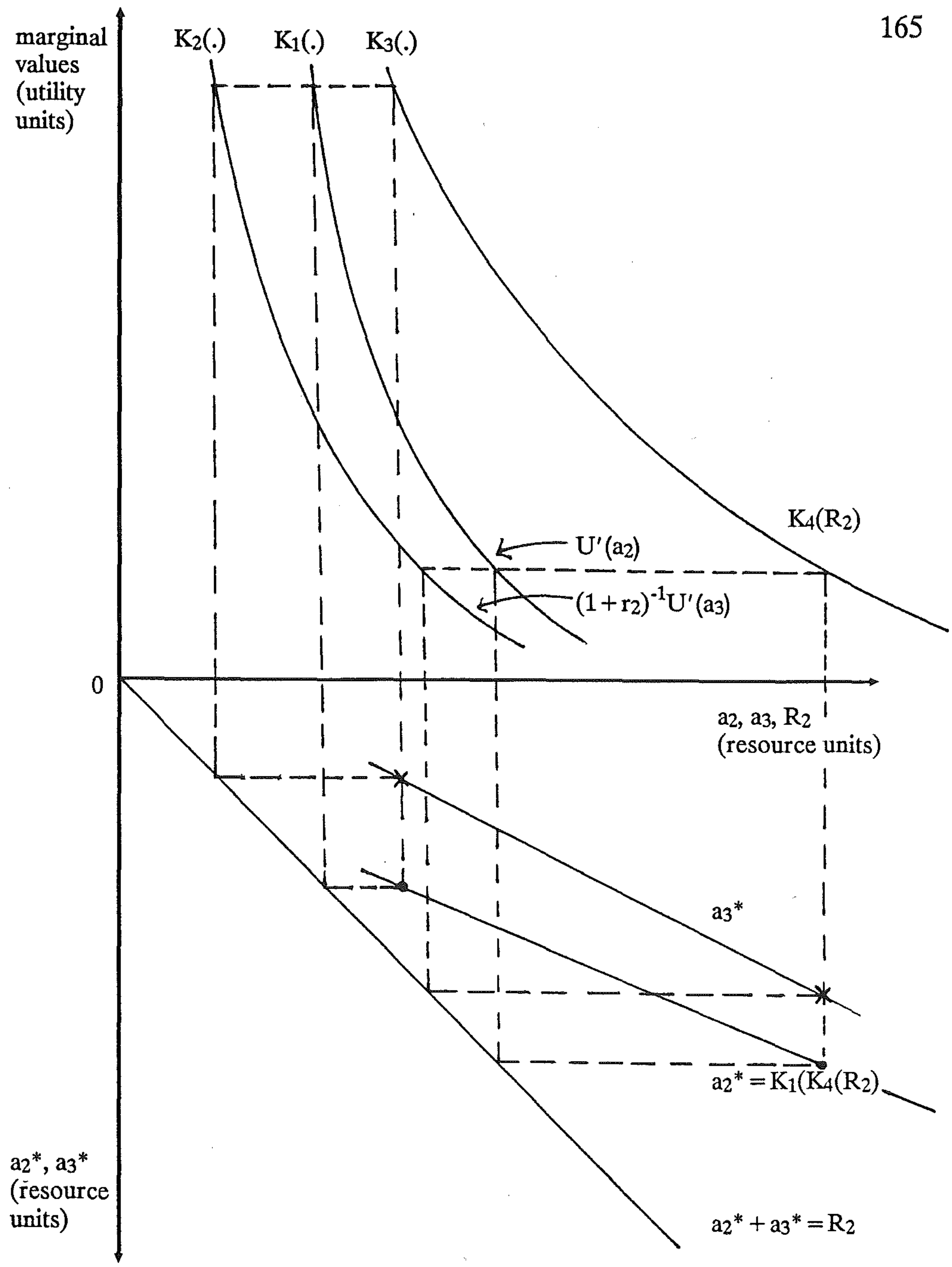


Figure 6.3: Optimal commitment as a function of resource availability

Similarly, the forecast of third period resource use a_3^*, for a given second period resource availability R_2, is:

$$a_3^* = K_2(K_4(R_2))$$

or equivalently:

$$a_3^* = R_2 - a_2^* = R_2 - K_1(K_4(R_2)) = R_2 - f_2(R_2)$$

The application of v_2 further thins the tree in Figure 6.1. After thinning, only branches with resource uses in the proportions ($M-R_2$, $f_2(R_2)$, $R_2 - f_2(R_2)$) remain viable. v_1 chooses between these branches.

$$v_1: \quad \max_{a_1} \; U(a_1) + \frac{1}{1+r_1} U(f_2(M-a_1)) + \frac{1}{(1+r_1)^2} U(M-a_1-f_2(M-a_1))$$

subject to: $\quad 0 \leq a_1 \leq M.$

Because the utility function ensures that the optimum a_1^* is interior, at optimality it is true that:

$$U'(a_1) = \frac{f_2'(M-a_1)}{1+r_1} U'(f_2(M-a_1)) + \frac{1-f_2'(M-a_1)}{(1+r_1)^2} U'(M-a_1-f_2(M-a_1))$$

It is convenient to replace M with the variable R_1 so as to find a general solution by the same procedure as before:

Define functions on the positive real line:

$$K_5 = \frac{f_2'(R_2)}{1+r_1} \quad U'(f_2(R_2)) + \frac{1-f_2'(R_2)}{(1+r_1)^2} \quad U'(R_2-f_2(R_2))$$

$:R_+ \rightarrow R_+$, the marginal valuation by the first period of resource left for later use,

$K_6 = K_5^{-1}$ $:R_+ \rightarrow R_+$, the inverse marginal valuation,

$K_7 = K_1 + K_6$ $:R_+ \rightarrow R_+$, the addition of the inverse valuations,

$K_8 = K_7^{-1}$ $:R_+ \rightarrow R_+$, inverse of the addition; i.e. the marginal worth of the resource R_1 to period 1.

Using the same reasoning as before, the initial optimal commitment a_1^* satisfies:

$$a_1^* = K_1(K_8(M)) = f_1(M)$$

Also,

$$R_2^* = K_5(K_8(M)).$$

The initial optimum a_1^* is derived just as for the second stage, with K_5 replacing $(1+r_2)^{-1}U'(.)$, and K_8 replacing K_4. Note that K_5 is a **revaluation**, by period one standards, of the commitments forecast to be adopted in periods two and three. K_5 therefore need not be related to the second periods valuation (K_4) of the same actions: in this model K_5

differs from K_4, by more than a proportional discount factor, whenever $r_2 \neq r_1$.

6.3 Suggestive results

A specific utility belief allows a more detailed investigation. The following increasing, strictly concave utility function, from the one parameter class of utility functions exhibiting constant relative risk aversion, satisfies the assumptions on utility listed above. Therefore all the required inverses exist. Also, the function has a convenient form for analytical manipulation.

Letting $U(a_t) = -a_t^{-1}, \quad 0 < a_t < \infty, \quad \text{all } t = 1,2,3.$

and adopting the positive solution for roots so the inverses are well-defined, gives:

$$U'(a_2) = a_2^{-2}, \qquad K_1(.) = (.)^{-1/2}$$

$$\frac{1}{1+r_2} U'(a_3) = \frac{1}{(1+r_2)a_3^2}, \qquad K_2(.) = ((1+r_2)(.))^{-1/2},$$

$$K_3(.) = (.)^{-1/2} + ((1+r_2)(.))^{-1/2} = (1+(1+r_2)^{-1/2})(.)^{-1/2},$$

$$K_4(R_2) = R_2^{-2}(1+(1+r_2)^{-1/2})^2,$$

$$K_1(K_4(R_2)) = R_2(1+(1+r_2)^{-1/2})^{-1},$$

therefore:

$$a_2^* = R_2.(1+(1+r_2)^{-1/2})^{-1} = f_2(R_2) = g_2(r_2).R_2.$$

As r_2 increases from zero towards infinity, $g_2(r_2)$, the fraction of inherited stock used in the second period, increases monotonically from 1/2 towards the limiting value of 1.0, as Figure 6.4 shows. That is, the second period uses a larger and larger fraction for itself as its discount rate increases - a result fully in accordance with the standard two-period cake-eating results. No divergence from these precommitment results is to be expected, because the two-period horizon prevents the difference between v_2 and v_3 from causing a difference between the a_3 period 2 sees as desirable and the a_3 period 3 adopts. A positive divergence from the precommitment results does occur when three periods are considered, as is shown below.

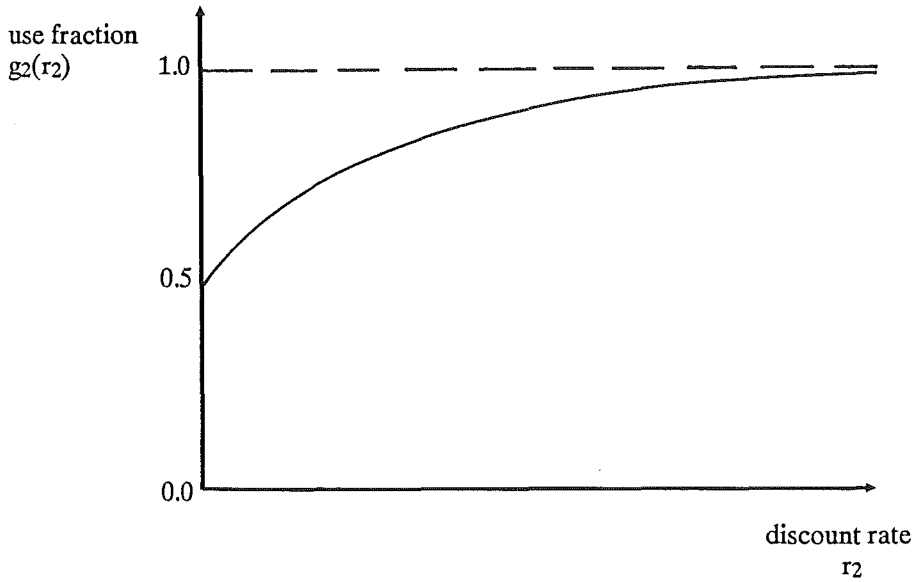


Figure 6.4: Second period use-fraction versus second period discount rate.

Restating:

$$a_2^* = f_2(R_2) = g_2(r_2).R_2.$$

Supressing the argument of g_2: $f_2' = g_2$, for all R_2,

therefore:

$$K_5(R_2) = \frac{g_2}{1+r_1}(g_2.R_2)^{-2} + \frac{1-g_2}{(1+r_1)^2}(R_2-g_2.R_2)^{-2}$$

$$= \left[\frac{1}{(1+r_1)g_2} + \frac{1}{(1+r_1)^2(1-g_2)}\right] R_2^{-2},$$

$$= hR_2^{-2}$$

$$K_6(.) = h^{1/2}(.)^{-1/2},$$

$$K_7(.) = K_6(.) + K_1(.) = h^{1/2}(.)^{-1/2} + (.)^{-1/2} = (1+h^{1/2})(.)^{-1/2},$$

$$K_8(R_1) = (1+h^{1/2})^2 R_1^{-2},$$

giving:

$$a_1^* = K_1(K_8(R_1)) = ((1+h^{1/2})^2 R_1^{-2})^{-1/2} = (1+h^{1/2})^{-1}.R_1$$

$$= f_1(R_1) = g_1(r_1,r_2).R_1$$

That is, for initial beliefs as assumed, it is optimal for the initial decision-maker to use a fraction g_1 of the initial stock. This fraction does not change with beliefs as to the size of the initial stock R_1. The fraction does however depend on beliefs about current **and** future attitudes to future welfare, as expressed in the discount rates r_1 and r_2. The function g_1 can therefore be interpreted as a belief-optimum mapping, as discussed in section 5.5.

The nature of this mapping is now explored. g_1 can be expressed in full as:

$$g_1(r_1,r_2) = [1+\{(1+r_1)^{-1}(1+(1+r_2)^{-1/2}) + (1+r_1)^{-2}(1+(1+r_2)^{1/2})\}^{1/2}]^{-1},$$

where all roots are interpreted as positive.

It is straightforward to show that:

$$0 \le g_1(r_1,r_2) < 1, \text{ for all } 0 \le r_1 < \infty, 0 \le r_2 < \infty,$$

$$g_1(0,0) = 1/3,$$

$$\lim_{r_1 \to \infty} g_1(r_1,r_2) = 1, \text{ for all } 0 \le r_2 < \infty$$

$$\lim_{r_2 \to \infty} g_1(r_1,r_2) = 0, \text{ for all } 0 \le r_1 < \infty$$

Also,

$$\frac{dg_1}{dr_1} = \frac{1}{2} g_1^2 h^{-1/2}(1+r_1)^{-1}(h+(1+r_1)^{-2}(1+(1+r_2)^{1/2})),$$

giving:

$$\frac{dg_1}{dr_1} > 0, \text{ for all } 0 \le r_1 < \infty, \; 0 \le r_2 < \infty,$$

$$\frac{dg_1}{dr_2} = \frac{1}{4} g_1^2 h^{-1/2} (1+r_2)^{-1/2} (1+r_1)^{-1} [(1+r_2)^{-1} - (1+r_1)^{-1}],$$

giving:

$$\frac{dg_1}{dr_2} \gtreqless 0 \quad \text{as} \quad r_1 \gtreqless r_2, \quad \text{for all } 0 \le r_1 < \infty, 0 \le r_2 < \infty.$$

Contours for g_1 in (r_1,r_2) - space are shown in Figure 6.5, which is not to scale.

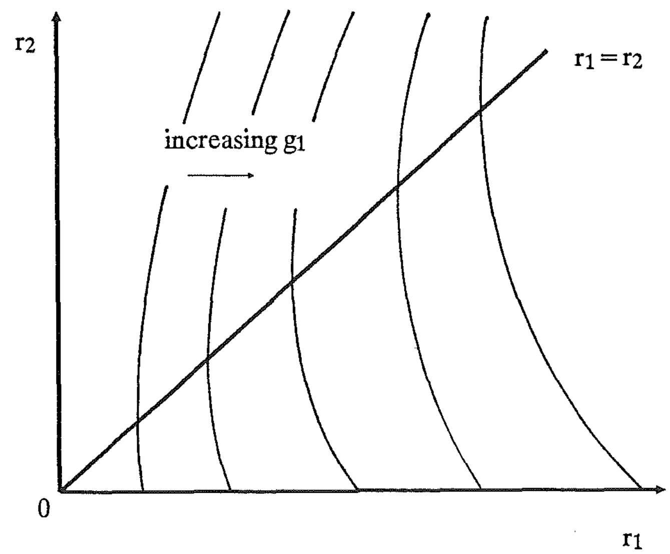


Figure 6.5: Contours for optimal initial fractional use in discount rate space

Knowledge of the function g_1 allows the set of commitment-forecast optima to be interpreted, and compared with the precommitment solution (for which $r_1 = r_2$):

- The precommitment optimum for a set discount rate involves a **higher** initial resource use than the commitment-forecast optimum for that initial rate followed by an increase **or** decrease in discounting. A precommitment approximation to a changing situation is, to this extent, biased towards initial resource use.

- A change in initial beliefs about the later discount rate changes the optimal initial use. Higher future discounting increases initial use if discount rates decline over time, and decreases initial use if discount rates increase over time.

- As in the precommitment case, it is always true that an increase in the initial discount rate increases initial resource use, provided everything else (including r_2 here) is held constant. However, if different periods' discount rates are related, so that a change in r_1 affects r_2, the 'speed-up' result may not hold. That is, a decrease in optimal initial use is possible from an increase in initial discounting if this also leads to an increase **or** decrease (of sufficient size) in later discounting.

6.4 Discussion

The recursive approach successfully derived the optimal commitment-forecast for a situation with changing choice procedures. The recursive structure ensured that this optimum is intertemporally consistent. Findings which go beyond the precommitment discoveries, and add to the existing knowledge about optimal non-renewable resource use, were obtained. The recursive approach therefore proved workable and worthwhile in one resource investigation.

The formulation employed is fairly restrictive. Temporally separable choice procedures, on the same utility function, with an invertible first derivative, are used. However, these same assumptions underpin those precommitment investigations which use constant-relative risk aversion utility. Also, the assumptions are convenient but not necessary to the existence of an optimum. They should be able to be weakened considerably in future work.

The results demonstrate that optimum non-renewable resource actions **are** sensitive to changes in future choice procedures, in a way not captured by the precommitment approximation. The finding that the precommitment optimum is biased towards the present, if discount rates increase **or** decrease over time, is particularly interesting. This shows that reliance on precommitment results can introduce a systematic error. Further exploration of this finding seems important.

Chapter Seven

Uncertain Choice Procedures

The previous chapter found that initial optimum resource use is sensitive to expected changes in choice procedures over time. These changes are in reality uncertain, because they are based on changes in individuals' tastes and ethical judgements, which are poorly understood. Non-renewable resource use may extend over a time period long enough for the uncertainty about future choice procedures to be significant: the uncertainty may affect which initial actions are optimal, as judged by initial choice procedures.

The precommitment formulations used to date cannot investigate this issue. They assume that there is only one choice procedure, applied by an initial decision-maker, who has the power to initially determine later actions. The recursive decision approach developed above is intended to allow uncertain future choice procedures to be investigated.

This chapter applies the recursive decision approach to a case involving uncertain future choice procedures and non-renewable resource depletion. This is done so as to test the workability of the recursive approach in uncertain contexts. Hopefully, any serious problems with the approach, which do not arise for the 'certain' case, such as that of the previous chapter, can then be identified. As with the latter case the intent is also to add to the theoretical base for resource use principles.

The application is not purely a test case. The importance of choice procedure uncertainty is to date unknown. The application may show whether and how this uncertainty impacts on optimal initial resource use, and may demonstrate that the assumption of certainty is or is not a good approximation.

The method adopted parallels that of the previous chapter. The formulation is extended to allow that the discount rates adopted by future decision-makers are initially uncertain. As before the three-period recursive structure is solved by backwards recursion, and the optimal initial action is derived. This is a function of the expected value of a function of the discount rates. The impact of a mean-preserving-spread in uncertainty can therefore be determined. Finally, the significance of the findings is briefly discussed.

7.1 Initial belief formulation

The three-period model developed in section 6.1 is now modified to allow each decision-maker to be uncertain about the discount rates which will be applied by future decision-makers.

To simplify the presentation two assumptions are made. Firstly, the uncertainty is captured by an initial probability distribution, for each period's discount rate, which is independent of previous choice procedures and actions. Secondly, all choice procedures are program risk-neutral; that is, they allow for uncertainty by using the expected value of an objective in place of its certain value.

Figure 7.1 illustrates initial beliefs of this type. The second choice procedure v_2 can be c_1 or c_2, and the third choice procedure v_3 can be c_3 or c_4. When only three periods are considered and only discount rates are uncertain then $c_3 = c_4$, since the v_3 do not involve discounting. The probabilities are not represented in Figure 7.1.

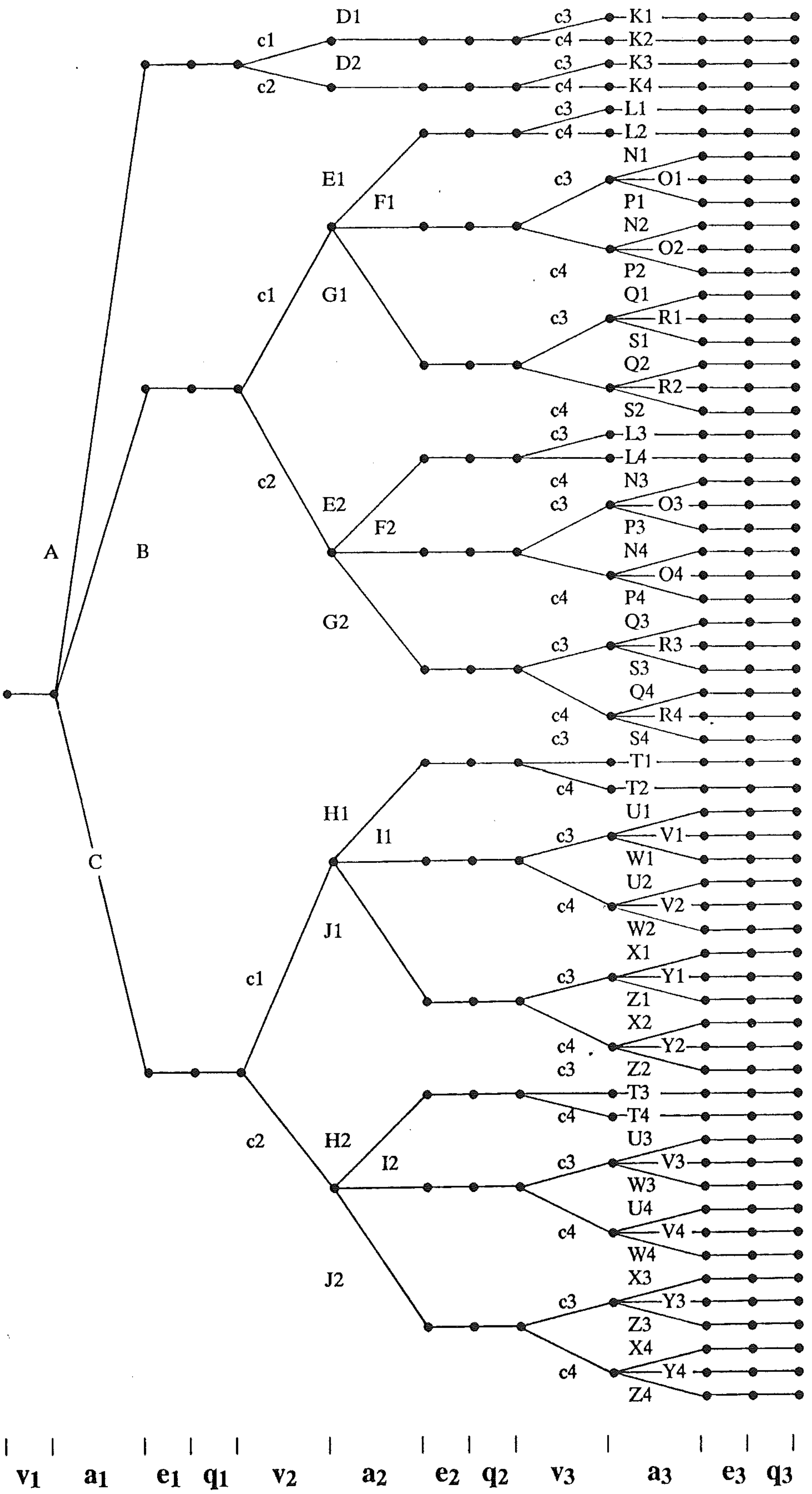


Figure 7.1: Three-period resource use with uncertain choice procedures

Actions are labelled with upper case. At decision nodes, the top branch action involves using the maximum available stock, while the bottom branch action involves zero use. Thus the action labels can be resource use measures. For example, if M is the maximum initial stock:

$$A = M, \qquad \text{and } D1 = D2 = K1 = K2 = K3 = K4 = 0.0$$

$$B + E1 = M, \quad \text{and } L1 = L2 = 0.0$$

Actions between the top and bottom branches, such as B, F1, and O1, represent the infinite set of branches emanating from decision nodes where the stock is not zero. Any course of action which entails positive use in all periods is in the set:

$$\{ (B,F1,O1), (B,F1,O2), (B,F2,O3), (B,F2,O4) \}.$$

Letting E denote the mathematical expectation operator, the initial beliefs can be represented by an unthinned recursive formulation:

v_3: (given a_1, a_2, M)

$$\max_{a_3} \; U(a_3)$$

subject to: $a_1 + a_2 + a_3 \leq M$

a_1, a_2, M given

$a_3 \geq 0$.

v_2: (given a_1, M, r_2)

$$\max_{a_2} \; U(a_2) + \frac{1}{1+r_2} U(a_3)$$

subject to: $a_1 + a_2 + a_3 \leq M$

a_3 solves v_3 (given a_1, a_2, M)

a_1, M given

$a_2 \geq 0$.

v_1: (given M)

$$\underset{a_1}{\text{Max}} \; \underset{r_2}{E} \left(U(a_1) + \frac{1}{1+r_1} U(b(r_2)) + \frac{1}{(1+r_1)^2} U(c(r_2)) \right)$$

subject to: $a_1 + b(r_2) + c(r_2) \leq M$, for all r_2
$b(r_2)$ solves v_2 (given a_1, M, r_2)
$c(r_2)$ solves v_3 (given a_1, $b(r_2)$, M)
M given
$a_1 \geq 0$.

7.2 Derivation of optimal initial action

Let the state variable R_t be the maximum resource use believed possible from period t onwards. The utility function $U(.):R_+ \rightarrow R$ is everywhere continuous and twice differentiable, with:

$$U'(.) > 0, \quad U'' < 0, \quad \lim_{a \to 0} U'(a) = \infty, \quad \lim_{a \to \infty} U'(a) = 0.$$

Also, $0 \leq r_1 < \infty$ and $0 \leq r_2 < \infty$ for every realization r_2 of the second period discount rate.

The period 3 optima are, as before, immediately available:

$$a_3^* = f_3(R_3) = R_3 = M\text{-}a_1\text{-}a_2 \text{ for all } a_1, a_2, M.$$

The second period discount rate is revealed before the second period choice is made, so for each r_2 and R_2, as before:

v_2: (given r_2, R_2)

$$\underset{a_2}{\text{Max}} \left(U(a_2) + \frac{1}{1+r_2} U(R_2 - a_2) \right)$$

subject to: $0 \leq a_2 \leq R_2$.

At optimality:

$$U'(a_2^*) = \frac{1}{1+r_2} U'(R_2 - a_2^*)$$

therefore:

$$a_2^* = (U')^{-1} \left[[(U')^{-1}\{.\} + (U')^{-1}\{(1+r_2)(.)\}]^{-1} \{R_2\} \right]$$

$$= K_1(K_4(R_2)) = f_2(R_2) = F_2(r_2,R_2)$$

In Figure 7.1 these optima are the only branches remaining after thinning the sub-tree following each v_2 node.

The period one problem is the first to face uncertainty. The problem can be rewritten using the formulae for the later optima:

v_1: (given R_1)

$$\underset{a_1}{\text{Max}} \ \underset{r_2}{E} \left(U(a_1) + \frac{1}{1+r_1} U(F_2(r_2,R_1-a_1)) + \frac{1}{(1+r_1)^2} U(R_1-a_1-F_2(r_2,R_1-a_1)) \right)$$

subject to: $0 \leq a_1 \leq R_1$.

At optimality:

$$U'(a_1) = \underset{r_2}{E}\left[\frac{\left.\frac{dF_2}{dR_2}\right|_{r_2,R_1-a_1}}{1+r_1}\,U'(F_2(r_2,R_1-a_1)) + \frac{1-\left.\frac{dF_2}{dR_2}\right|_{r_2,\,R_1-a_1}}{(1+r_1)^2}\,U'(R_1-a_1-F_2(r_2,R_1-a_1))\right]$$

The right-hand-side is the expected marginal value, to the first period objective, of resource left for future use ($R_2 = R_1 - a_1$). For each given r_1 and realisation r_2 there is a 'revaluation' curve $K_5(R_2)$ as in Chapter 6. For a given probability distribution on r_2 the 'expected revaluation' curve $K_9(R_2)$ can be formed:

$$K_9(R_2) = \underset{r_2}{E}\left(K_5(R_2) \mid r_2\right)$$

This is illustrated in Figure 7.2, where there are two equiprobable future discount rates, r_2^1 and r_2^2.

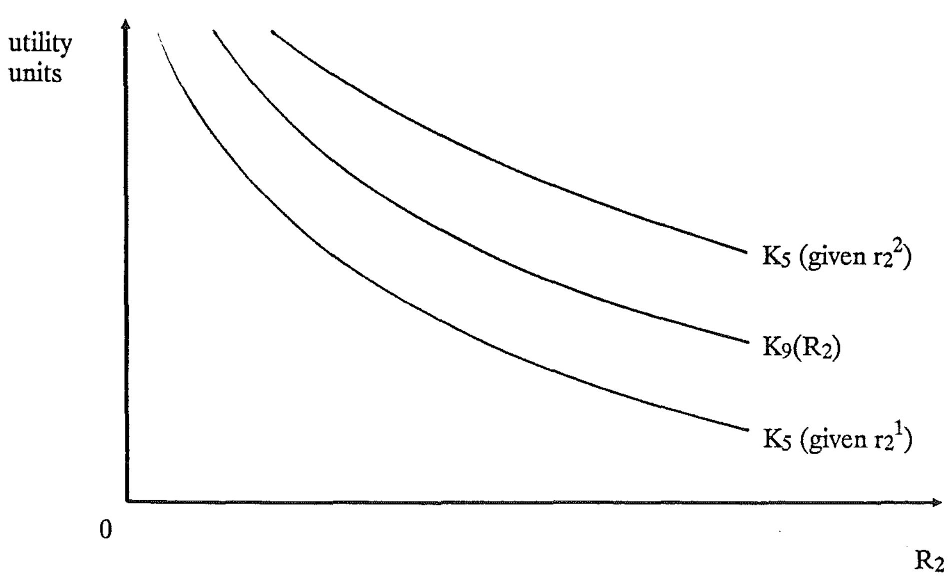


Figure 7.2: Expected value of unused resources

Continuing as before, the first period optimum can be obtained by inversion and summation of the marginal value curves:

$K_{10}(.) = (K_9)^{-1}(.)$ $\quad :R_+ \rightarrow R_+,$ the future resource use level which brings about a marginal value level.

$K_{11}(.) = K_{10}(.) + K_1(.)$ $\quad :R_+ \rightarrow R_+,$ the total resource use which brings about a marginal value level.

$K_{12}(R_1) = (K_{11})^{-1}(R_1)$ $\quad :R_+ \rightarrow R_+,$ the marginal value level resulting from set total resource availability.

Therefore:

$$a_1^* = K_1(K_{12}(M)) .$$

7.3 Suggestive results

The specific utility belief employed in Chapter 6 is used again to provide solutions simple enough for analytical investigation:

$$U(a_t) = -a_t^{-1}, \quad 0 < a_t < \infty, \quad t = 1,2,3.$$

Throughout the following roots are interpreted as positive, in accordance with the function definitions, so that inverse functions are well defined over the positive real line wherever required.

As before,

$$U'(a_2) = a_2^{-2},$$

$$K_1(.) = (.)^{-1/2}$$

$$K_4(R_2) = R_2^{-2}\ [1 + (1+r_2)^{-1/2}\ ]^2$$

Therefore:

$$a_2^* = K_1(K_4(R_2)) = F_2(r_2,R_2) = [1+(1+r_2)^{-1/2}]^{-1} .R_2$$

$$a_3^* = [1+(1+r_2)^{1/2}]^{-1} .R_2$$

Recalling that r_2 is now a random variable,

$$K_9(R_2) = \underset{r_2}{E}\left[(1+r_1)^{-1} \left.\frac{dF_2}{dR_2}\right|_{r_2} (F_2(r_2,R_2))^{-2} + (1+r_1)^{-2}\left[1-\left.\frac{dF_2}{dR_2}\right|_{r_2}\right](R_2-F_2(r_2,R_2))^{-2}\right]$$

$$= \underset{r_2}{E}\left[\left\{ (1+r_1)^{-1}(1+(1+r_2))^{-1/2} + (1+r_1)^{-2}(1+(1+r_2)^{-1/2}) \right\} .R_2^{-2} \right]$$

$$= [\underset{r_2}{E}\ \{h(r_1,r_2)\}] .R_2^{-2}$$

$$= j .R_2^{-2}.$$

Therefore:

$$K_{10}(.) = j^{1/2}\ (.)^{-1/2}$$

$$K_{11}(.) = (1+j^{1/2})\ (.)^{-1/2}$$

$$K_{12}(R_1) = (1+j^{1/2})^2 R_1^{-2}$$

so:

$$a_1^* = (1+j^{1/2})^{-1}.R_1 = \left[1 + [E\ \{ h(r_1, r_2) \}\ ]^{1/2}\ \right]^{-1} . R_1$$

The sensitivity of a_1^* to the uncertainty about r_2 depends on the nature of $h(r_1,r_2)$. It is easily shown that:

$$h(r_1,r_2) > 0, \quad \text{for all} \quad 0 \le r_1 < \infty, \quad 0 \le r_2 < \infty.$$

$$h(0,0) = 4$$

$$\lim_{r_1 \to \infty} h(r_1,r_2) = 0, \quad \text{for all } 0 \le r_2 \le \infty$$

$$\lim_{r_2 \to \infty} h(r_1,r_2) = \infty, \quad \text{for all } 0 \le r_1 < \infty$$

Further,

$$\frac{dh}{dr_2} = \frac{1}{2}[\,(1+r_1)^{-2}(1+r_2)^{-1/2} - (1+r_1)^{-1}(1+r_2)^{-3/2}\,]$$

$$= \frac{1}{2}\,(1+r_1)^{-2}(1+r_2)^{-3/2}(r_2 - r_1).$$

Therefore,

$$\frac{dh}{dr_2} \ \substack{> \\ = \\ <}\ 0 \quad \text{as} \quad r_2 \ \substack{> \\ = \\ <}\ r_1.$$

Also,

$$\frac{d^2h}{dr_2} = \frac{1}{4}[\,3\,(1+r_1)^{-1}(1+r_2)^{-5/2} - (1+r_1)^{-2}(1+r_2)^{-3/2}\,]$$

$$= \frac{1}{4}\,(1+r_1)^{-2}\,(1+r_2)^{-5/2}\,\left(3r_1 - (r_2+2)\right),$$

$$\text{so } h \text{ is locally } \begin{array}{l}\text{convex}\\ \text{inflecting}\\ \text{concave}\end{array} \text{ in } r_2, \quad \text{as } \frac{d^2h}{dr_2^2} \begin{array}{c}>\\=\\<\end{array} 0, \quad \text{as } r_1 \begin{array}{c}>\\=\\<\end{array} \frac{r_2-2}{3}.$$

Figure 7.3, which is not drawn to scale, depicts the shape of $h(r_1,r_2)$. Note that h is convex in r_2, for all r_1, over the range $0 \le r_2 \le 2$.

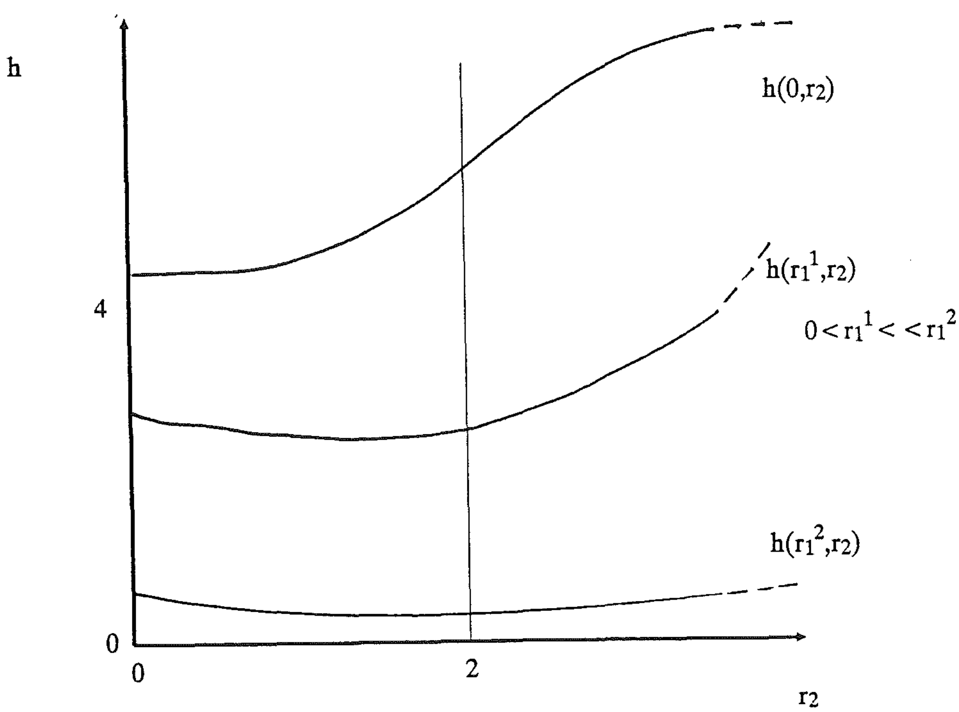


Figure 7.3: The impact of uncertain discount rates

The Appendix demonstrates the use of mean-preserving-spreads (mps) in conjunction with Jensen's inequality to derive the sensitivity of an identity to uncertainty. This is applied here by noting that a_1^* is a function of

$$\underset{r_2}{E}\,[\,h(r_1, r_2)\,]\,.$$

If $h(r_1,r_2)$ is convex in r_2,

$$\underset{r_2}{E}\,[\,h(r_1, r_2)\,] \qquad \text{increases with mps in } r_2,$$

$$\text{and} \quad +\sqrt{\underset{r_2}{E}\,[\,(h(r_1, r_2)\,]} \qquad \text{increases with mps in } r_2,$$

$$\text{and} \quad \left(1 + [\underset{r_2}{E}\,(\,h\,(r_1, r_2)\,)\,]^{1/2}\right) \qquad \text{decreases with mps in } r_2.$$

That is, for $0 \leq r_2 \leq 2$, an increase in the uncertainty about r_2 means that the optimum initial resource use is reduced.

This range for r_2 certainly captures the utility discount rates generally used in the literature, which commonly have a value of 0.1. However, if a very long intergenerational period is what is approximated by the three periods considered here, then discount rates which reflect usual planning procedures (with horizons of fifteen years at maximum) would be much higher - certainly of the same order of magnitude as 2. The sensitivity of a_1^* to uncertain r_2 then requires more detailed investigation, and in particular specific probability distribution assumptions must be made.

7.4 Discussion

The recursive approach successfully derived an intertemporally consistent optimal commitment-forecast, for an initial belief that future choice procedures are uncertain. As in the previous chapter, the findings go beyond existing precommitment discoveries, to add to the knowledge of resource depletion. Therefore, the tractability of the recursive approach extends to at least one uncertain context.

For the specific case investigated, the appropriate initial non-renewable resource use is sensitive to uncertainty about future choice procedures, in a way not captured by deterministic approximations, such as optimising for the expected situation.

In particular, uncertainty about later discount rates reduces optimal initial resource use for a well-defined range of later discount rates including those generally assumed. The impact of any specific uncertainty can be derived (by comparison with the mean case) because an exact solution for optimal resource use is available.

A further step in analysis of uncertainty would be to seek an analytical representation of the difference between the mean and uncertain cases. This requires that further assumptions on the form of the uncertainty be made.

It is straightforward to further extend the procedure to allow for uncertainties which depend on preceding happenings, for further periods, and for multiple sources of uncertainty.

Chapter Eight

Resolution Timing

The two preceding chapters have demonstrated that the recursive approach has potential in the analysis of decisions in dynamic, uncertain contexts. In such contexts many decisions concern research activities, undertaken to reduce uncertainty.

Research influences the timing of uncertainty resolution. Research provides a basis for predicting events which will eventually be observable, so no longer uncertain. The prediction is therefore a contribution toward the resolution of uncertainty. Research results also affect which actions are undertaken, hence which events may be observed, hence the timing of resolution of uncertainty.

An earlier resolution of uncertainty is often valuable because, for many objectives, it allows better actions to be identified and adopted. Early knowledge of the long-run availability of non-renewable resources and their substitutes is necessary to avoid the problems of over- or under-use in both decentralized and centralized contexts.

Some research directed at earlier resolution of uncertainty appears to be undertaken for reasons other than potential implications for action. Knowledge of the ultimate fate of the universe seems to fall into this category. In this case the preference for early resolution outweighs the 'worsening' of consequences implied by the use of resources with opportunity costs.

If the prevailing choice procedure is concerned about the uncertainty experienced in future situations, resolution timing is important. Its importance here is irrespective of any impact on action, as Figure 8.1 illustrates.

The numbers in the Figure are the event probabilities. The later actions (C, D, E, F, G) have been forecast, enabling the initial decision-maker to face a thinned tree depicting the consequences of choice between A and B. The attributes of concern to the decision-maker take on values a, b, c, d, e. The consequences of A and B differ only by their resolution timing, the only uncertain situation which may be experienced is after B. If this situation is not important to the decision-maker then A and B are equally appropriate actions.

The usual choice procedure, of maximizing the expected value of the utility of a time-stream of attributes, cannot incorporate attitudes toward uncertain situations. No such procedure can differentiate between A and B. To do so, the approach must be generalized: each possible initial outcome of initial choice is treated as a structured sub-tree, not just a probability distribution over a set of branches.

This chapter applies the recursive approach to non-renewable resource problems where resolution timing is important. The investigation seeks to separate the impact of resolution timing on actions from its possible importance in formulating choice procedures, and to clarify the implications of resolution timing for intertemporal consistency.

The method adopted is to modify the formulation used in previous chapters, to cover uncertainty about technology in the third period. The optimal commitment-forecasts, for two different timings of uncertainty resolution, are then derived and compared. Discussion of the significance of the findings concludes the chapter.

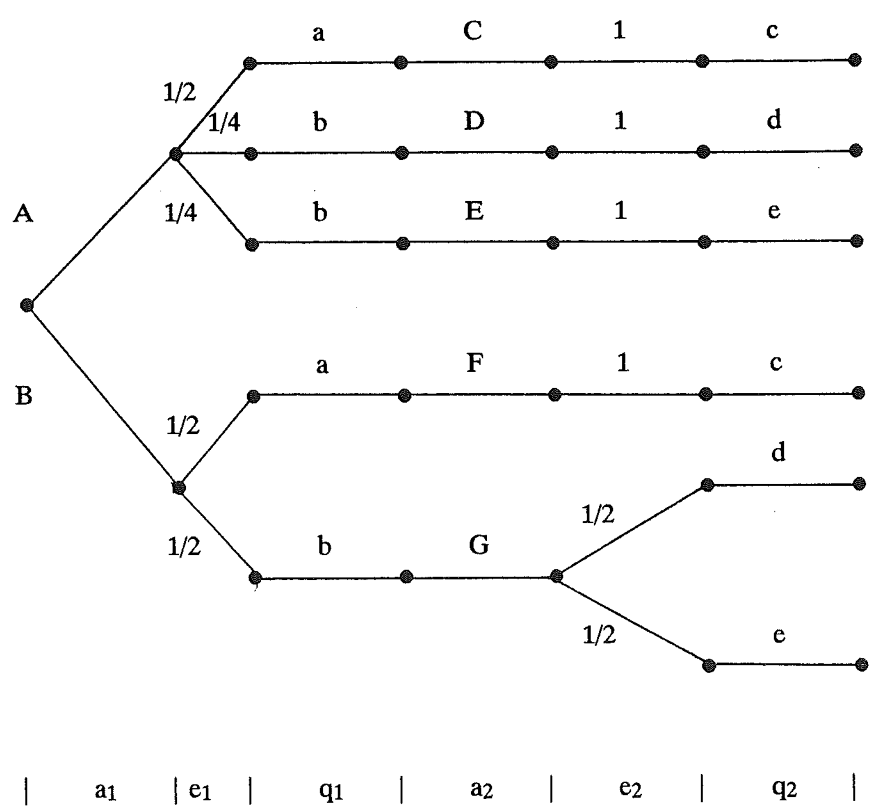


Figure 8.1: Resolution timing independence

8.1 Initial belief formulation

The impact of the timing of uncertainty resolution on actions can be investigated for modified versions of the three period depletion model introduced in Chapter 6.

The first modification is for it to be believed that in the third period an improvement in resource recovery will be made, and its size will be unknown until then. In the second modification the size of the third period change becomes known before second period choice, so uncertainty is resolved earlier.

8.1.1 Late resolution

Let the initial and following beliefs be as outlined in Chapter 6, with the addition that it is believed that a technological improvement will increase resource recovery by a factor $K \geq 1$ in the third period. The initial and second period (prior) belief is that the probability density governing K is:

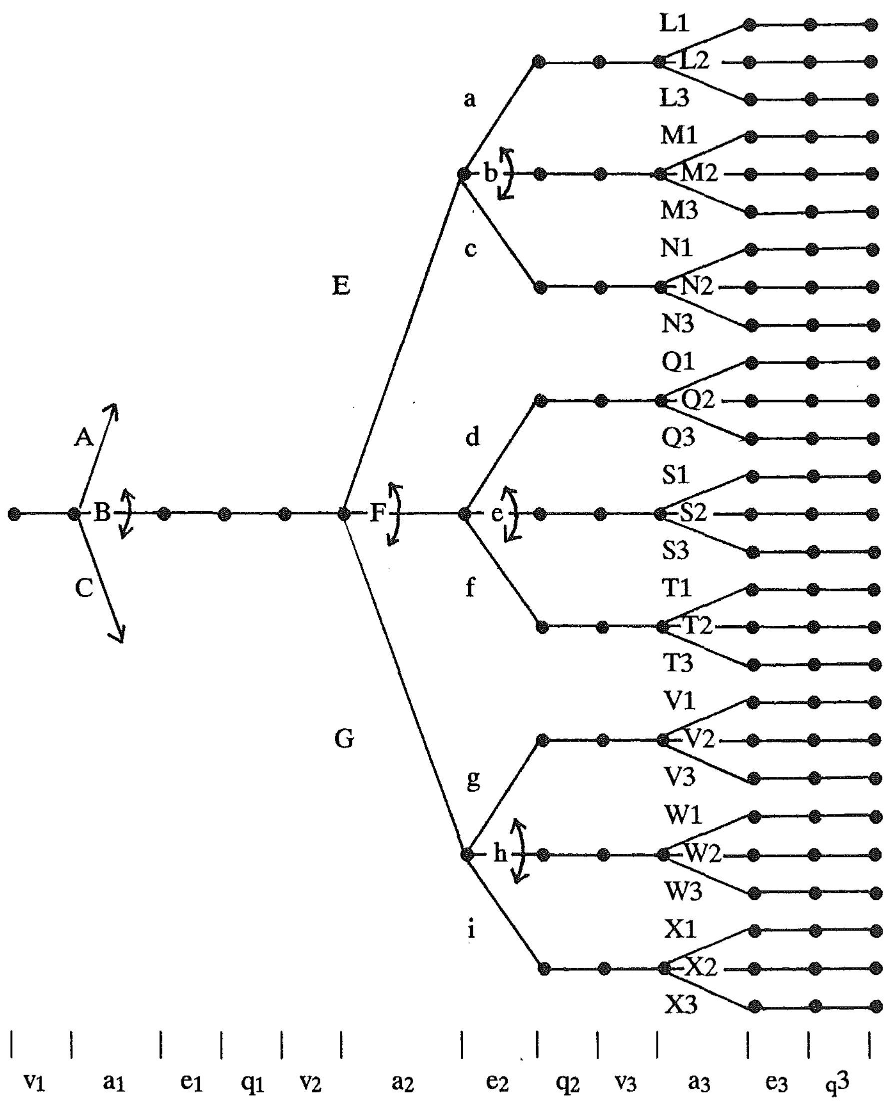
$$P(K): [1, \infty] \rightarrow [0, \infty]$$

so that

$$\int_0^\infty P(K) \ dK = 1.$$

The third period faces no uncertainty, and the second and first decision-makers maximize the expected value of their previous certain objectives.

All decision-makers are therefore neutral towards the timing of resolution of uncertainty about the utility streams. Figure 8.2 depicts the situation in (unthinned) tree form.



Key to labels:

A and C are upper and lower bounds on initial action, B is a 'typical' interior action.

E and G are upper and lower bounds on period two action, F is a 'typical' interior action given B.

a, d, g and c, f, i are the upper and lower bounds on the size of the factor K.

S1 and S3 are upper and lower bounds on period 3 resource use, and S2 is an interior action, given actions B and F and event e. Other period 3 actions vary similarly.

Figure 8.2: Uncertainty about third period events

The recursive formulation is:

v_3: (given a_1, a_2, M, K)

$$\max_{a_3} U(a_3)$$

subject to: $a_3 \leq K(M-a_1-a_2)$
a_1, a_2, M, K given
$a_3 \geq 0$.

v_2: (given a_1, M)

$$\max_{a_2} \underset{K}{E} \left(U(a_2) + \frac{1}{1+r_2} U(b(a_1, a_2, M, K)) \right)$$

subject to: $a_2 \leq M-a_1$
b solves v_3 for a_1,a_2,M,K
a_1,M given
$a_2 \geq 0$.

v_1: (given M)

$$\max_{a_1} \underset{K}{E} \left(U(a_1) + \frac{1}{1+r_1} U(c(a_1,M)) + \frac{1}{(1+r_1)^2} U(b) \right)$$

subject to: $a_1 + c(a_1, M) \leq M$
$c(a_1,M)$ solves v_2 for a_1,
b solves v_3 for a_1, $c(a_1, M)$, M, K
M given
$a_1 \geq 0$.

8.1.2 Derivation of optimal initial action

Let R_t be a state variable expressing the use of resource possible from t onwards, and thereby reflecting previous actions. Let $0 \leq r_1 < \infty$ and $0 \leq r_2 < \infty$, and the realisations of K, and hence its expected value, be bounded above.

$U(.): R_+ \rightarrow R$ is everywhere continuous and twice differentiable, with

$$U' > 0, \qquad U'' < 0 \qquad \lim_{a \rightarrow 0} U'(a) = \infty \qquad \lim_{a \rightarrow \infty} U'(a) = 0$$

The period three optima are therefore immediately available:

$$a_3^* = K(M - a_1 - a_2) = KR_3, \quad \text{for all } a_1,\ a_2,\ M,\ K.$$

This is used to define $b(K, a_1, a_2, M) = b(K, R_3) = KR_3$.

The second period faces uncertainty about K:

v_2: (given $R_2 = M - a_1$)

$$\max_{a_2} J_2 = \mathop{E}_{K} \left(U(a_2) + \frac{1}{1+r_2} U(b(K, R_3)) \right)$$

subject to:

$$a_2 \leq R_2$$
$$R_3 = R_2 - a_2$$
$$a_2 \geq 0.$$

The conditions on the utility function ensure that at optimality

$$\frac{dJ_2}{da_2} = 0,$$

that is, there is an interior optimum.

Therefore:

$$U'(a_2) + \frac{1}{1+r_2} \frac{d}{da_1} \underset{K}{E} \left[U(K(R_2 - a_2)) \right] = 0$$

$$\Rightarrow \quad U'(a_2) = \frac{1}{1+r_2} \underset{K}{E} \left[KU'(K(R_2 - a_2)) \right] = \frac{1}{1+r_2} \underset{K}{E} \left[KU'(KR_3) \right]$$

As in previous chapters the RHS of this equation can be written as a function of R_3:

$$K_5(R_3) = \frac{1}{1+r_2} \underset{K}{E} \left[KU'(K(R_3)) \right] \qquad : R_+ \rightarrow R_+,$$

deriving:

$$K_6(.) = K_5^{-1} (.) \qquad : R_+ \rightarrow R_+.$$

The horizontal addition of the marginal utility of period two use (LHS) and later use (RHS) uses, as previously,

$$K_1(.) \equiv (U')^{-1}(.) \qquad : R_+ \rightarrow R_+;$$

giving

$$K_7(.) = K_1(.) + K_6(.) \qquad : R_+ \rightarrow R_+$$

so

$$K_8(R_2) = K_7^{-1} (R_2) \qquad : R_+ \rightarrow R_+.$$

The function $K_8(R_2)$ is the marginal utility to the second decision-maker of resource available at the start of period two.

Therefore,

$$a_2^* = K_1(K_8(R_2)) = F_2(R_2).$$

Knowledge of a_3* and a_2* enables thinning of the tree in Figure 8.2 at each v_3 node and then each v_2 node. Also, for use at v_1:

$$c(a_1, M) = F_2(M\text{-}a_1) = F_2(R_2).$$

Rewriting the initial decision-maker's problem:

v_1: (given R_1)

$$\underset{a_1}{\text{Max}} \; \underset{K}{E} \left(U(a_1) + \frac{1}{1+r_1} U(F_2(R_1\text{-}a_1)) + \frac{1}{(1+r_1)^2} U[K(R_1\text{-}a_1\text{-}F_2(R_1\text{-}a_1))] \right)$$

subject to: $\quad 0 \le a_1 \le R_1,$

and at optimality:

$$U'(a_1) + \frac{d}{da_1} \underset{K}{E} \left(\frac{1}{1+r_1} U(F_2(R_1\text{-}a_1)) + \frac{1}{(1+r_1)^2} U[K(R_1\text{-}a_1\text{-}F_2(R_1\text{-}a_1))] \right) = 0.$$

The function F_2 incorporates the expected value of a function of K in its parameters, so it is invariant with the realization of K employed in forming the expectation immediately above.

The expected marginal value K_9 (to the initial decision-maker) of later use R_2 can be formed as the expectation of the marginal value given each realisation K.

Therefore, at optimality:

$$U'(a_1) = K_9(R_2) = K_9(R_1\text{-}a_1).$$

As before, the inverse of the addition of the inverses of the LHS and RHS gives the expected value $K_{12}(R_1)$ (to the initial decision-maker) of resource R_1 available at the beginning of the initial period.

So it follows that:

$$a_1^* = K_1(K_{12}(M)).$$

8.1.3. Suggestive results

Let $U(a_t) = -a_t^{-1}$, $0 < a_t < \infty$, $t = 1,2,3$, so that simple analytical results can be derived. Interpreting all roots as positive, so that the required inverse functions everywhere exist:

$$a_3^* = K(R_2\text{-}a_2).$$

Therefore at v_2 optima:

$$a_2^{-2} + \frac{d}{da_2}\left(\mathop{E}_{K} \left[\frac{1}{1+r_2} .\text{-} K^{-1} (R_2\text{-}a_2)^{-1} \right] \right) = 0$$

$$\Rightarrow \quad a_2^{-2} = (1+r_2)^{-1} . \mathop{E}_{K} (K^{-1}) . (R_2\text{-}a_2)^{-2}$$

$$\Rightarrow \quad a_2 \quad = \quad (1+r_2)^{1/2} \, (\underset{K}{E} \, [K^{-1}])^{-1/2} (R_2 - a_2)$$

$$\Rightarrow \quad a_2 \quad = \Big[1+(1+r_2)^{1/2} (\underset{K}{E}[K^{-1}])^{-1/2}\Big]^{-1} (1+r_2)^{1/2} \, [\underset{K}{E} \, (K^{-1})]^{-1/2} .R_2.$$

$$\text{Let } \; Y \quad = \Big[1+(1+r_2)^{1/2} \, (\underset{K}{E}[K^{-1}])^{-1/2}\Big]^{-1} (1+r_2)^{1/2} \, [\underset{K}{E} \, (K^{-1})]^{-1/2}$$

$$= \Big[1 + (1+r_2)^{-1/2} \, (\underset{K}{E} \, [K^{-1}])^{1/2}\Big]^{-1} .$$

Then the initial problem can be written:

v_1:

$$\underset{a_1}{\text{Max}} \; \underset{K}{E} \left(U(a_1) + \frac{1}{1+r_1} U(Y(R_1-a_1)) + \frac{1}{(1+r_1)^2} U\,(K\,(1\text{-}Y)\,(R_1-a_1)) \right)$$

subject to: $\quad 0 \leq a_1 \leq M.$

Allowing $U(.) = -(.)^{-1}$, therefore, gives:

v_1:

$$\underset{0 \leq a_1}{\text{Max}} \; \underset{K}{E} \Big[-a_1^{-1} + (1+r_1)^{-1} . - (Y(R_1-a_1))^{-1} + (1+r_1)^{-2} . - (K.(1\text{-}Y)\,(R_1-a_1))^{-1} \Big]$$

$$= \underset{0 \leq a_1}{\text{Max}} \; -a_1^{-1} - \underset{K}{E} \Big[(R_1-a_1)^{-1} [(1+r_1)^{-1} Y^{-1} + (1+r_1)^{-2} (1\text{-}Y)^{-1} K^{-1}] \Big]$$

$$= \max_{0 \leq a_1} -a_1^{-1} - (R_1-a_1)^{-1} \left[(1+r_1)^{-1} Y^{-1} + (1+r_1)^{-2} (1-Y)^{-1} \underset{K}{E} [K^{-1}] \right]$$

At optimality $\dfrac{d}{da_1} = 0$, therefore:

$$a_1^{-2} = (R_1-a_1)^{-2} \left[(1+r_1)^{-1} Y^{-1} + (1+r_1)^{-2} (1-Y)^{-1} \underset{K}{E} [K^{-1}] \right].$$

Expanding Y gives:

$$a_1^{-2} = (R_1-a_1)^{-2} \Big[(1+r_1)^{-1} (1+ (\underset{K}{E} [K^{-1}])^{1/2} (1+r_2)^{-1/2})$$
$$+ (1+r_1)^{-2} (1+ (1+r_2)^{1/2} (\underset{K}{E} [K^{-1}])^{-1/2}) \underset{K}{E} [K^{-1}] \Big]$$

$$= (R_1-a_1)^{-2} \Big[(1+r_1)^{-2} \underset{K}{E} [K^{-1}] + (1+r_1)^{-1}$$
$$+ ((1+r_1)^{-1} (1+r_2)^{-1/2} + (1+r_1)^{-2} (1+r_2)^{1/2} (E [K^{-1}])^{1/2} \Big]$$

Defining W by:

$$a_1^{-2} = (R_1-a_1)^{-2} .W$$

$$\Rightarrow a_1 = (R_1-a_1) W^{-1/2} = (1+W^{-1/2})^{-1} W^{-1/2} .R_1 = (1+W^{1/2})^{-1} R_1$$

$$\Rightarrow a_1^* = (1+W^{1/2})^{-1} M.$$

This equation for a_1 allows the sensitivity of the depletion decision, to changes in the parameters, to be investigated. In particular the response to a mean-preserving-spread in uncertainty about K can be investigated. To estimate sensitivity to the timing of uncertainty resolution, however, the following further developments are required.

8.2 Early resolution

The formulation of section 8.1 is now modified to reflect early resolution of third period uncertainty. The adjustment factor K is now **discovered** at the start of the second period, but still not **used** until the third period. The second decision-maker now faces no uncertainty. Figure 8.3 depicts this situation.

8.2.1 Formulation

The nested problems are:

v_3: (given $R_3 = M - a_1 - a_2, K$)

$$\max_{a_3} \; U(a_3)$$

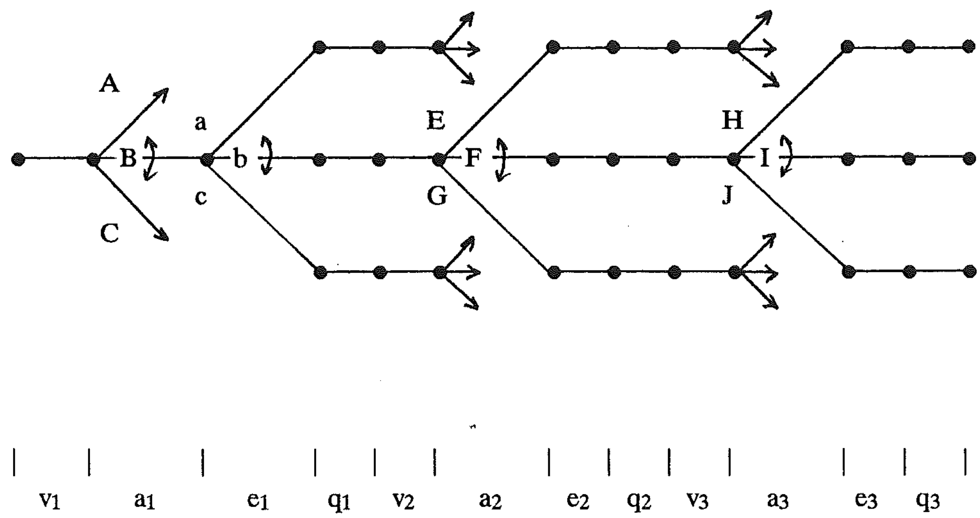
subject to: $\quad 0 \leq a_3 \leq KR_3$.

v_2: (given $R_2 = M - a_1, K$)

$$\max_{a_2} \; U(a_2) + \frac{1}{1+r_2} U(d(R_3,K))$$

subject to: $\quad d(R_3,K)$ solves v_3 for $(R_3 = R_2 - a_2, K)$

$\qquad 0 \leq a_2 \leq R_2$.



Key to labels:

A and C are upper and lower bounds on initial action, and B is a typical interior action.

Similarly for E, F, G and H, I, J in periods 2 and 3.

a and c are upper and lower bounds on the realization of the size of K, which occurs after the initial decision and before the period two decision.

Figure 8.3: Early resolution of third period uncertainty

v_1: (given M)

$$\underset{a_2}{\text{Max}} \; \underset{K}{E} \left[U(a_1) + \frac{1}{1+r_1} U(F(R_2, K)) + \frac{1}{(1+r_1)^2} U(d(R_3,K)) \right]$$

subject to:

$$R_2 = M - a_1$$
$$R_3 = R_2 - F(R_2, K)$$
$F(R_2, K)$ solves v_2 for R_2, K;
$d(R_3, K)$ solves v_3 for R_3, K;
$$0 \leq a_2 \leq M$$

Note that, through their dependence on K, the functions F and d and the state variable R_3 are initially viewed as random variables.

8.2.2 Derivation of optimal initial action

The procedure closely parallels that for late resolution. The third period decision function is available by inspection:

$$a_3^* = KR_3.$$

This defines

$$d(K, R_3) = KR_3 = K(R_2 - a_2)$$

The second period faces no uncertainty,

v_2 (given R_2,K):

$$\underset{a_2}{\text{Max}} \left[U(a_2) + \frac{1}{1+r_2} U(K(R_2 - a_2)) \right]$$

subject to: $0 \leq a_2 \leq R_2$.

At optimality:

$$U'(a_2) = \frac{K}{1+r_2} U'(K(R_2-a_2))$$

As previously, inverting each side, summing and reinverting gives the marginal value function $K_{13}(.)$, for the second decision-maker, of R_2: this time with the additional parameter K.

Therefore

$$a_2^* = K_1(K_{13}(R_2)) \equiv F(R_2,K).$$

So far, the a_3^* and a_2^* optima permit the tree in Figure 8.2 to be thinned back to the v_2 nodes. The initial decision problem is:

v_1:

$$\underset{a_1}{\text{Max}} \; \underset{K}{E} \left[U(a_1) + \frac{1}{1+r_1} U(F(M-a_1,K)) + \frac{1}{(1+r_1)^2} U(K(M-a_1-F(M-a_1,K))) \right] = 0$$

subject to: $0 \leq a_1 \leq M$,

and at optimality:

$$U'(a_1) + \frac{d}{da_1} \underset{K}{E} \left[\frac{1}{1+r_1} U(F(M-a_1,K)) + \frac{1}{(1+r_1)^2} U(K(M-a_1-F(M-a_1,K))) \right] = 0.$$

Provided the differentiation and expectation operations can be exchanged the previous approach can be applied. The marginal value of future use, as a function of carried over resource, can be found for each realisation of K, allowing formation of the expected marginal value function. This can be inverted, summed with the inverse of the initial period marginal utility function, and reinverted, giving the expected marginal value $K_{14}(.)$ as a function of the initial stock R_1.

It follows that:

$$a_1^* = K_1(K_{14}(M)).$$

Alternatively, a direct formation of the total value of future use, as a function of carried over resource, and the realisation of K, is required. The expectation over K can then be differentiated to solve the identity. As throughout this work, the iterative approach allows ready use of numerical approximations where analytical forms are unavailable.

8.2.3. Suggestive results

As in section 8.1.3, let

$$U(a_t) = -a_t^{-1}, \quad 0 < a_t < \infty. \quad t = 1,2,3.$$

Simple analytical results follow provided all roots are interpreted as positive so that the required inverse functions everywhere exist.

$$a_3^* = K(R_2-a_2)$$

At v_2 optima:

$$U'(a_2) + \frac{1}{1+r_2} . -K\, U'(K(R_2-a_2)) = 0$$

$$\Rightarrow \quad a_2^{-2} = (1+r_2)^{-1} K \;\; K^{-2} (R_2-a_2)^{-2}$$

$$\Rightarrow \quad a_2 \quad = \quad (1+r_2)^{1/2} K^{1/2} (R_2 - a_2)$$

$$\Rightarrow \quad a_2^* \quad = \quad (1+(1+r_2)^{1/2} K^{1/2})^{-1} (1+r_2)^{1/2} K^{1/2} R_2$$

$$\text{Let} \quad Z(K) = (1+(1+r_2)^{1/2} K^{1/2})^{-1} (1+r_2)^{1/2} K^{1/2}$$

$$= (1+(1+r_2)^{-1/2} K^{-1/2})^{-1}.$$

Then the initial problem can be written:

v_1:

$$\underset{0 \le a_1 \le M}{\text{Max}} \ \underset{K}{E} \Big[U(a_1) + \frac{1}{1+r_1} U(Z(K)(M-a_1)) + \frac{1}{(1+r_1)^2} U(K(1-Z(K))(M-a_1)) \Big]$$

Substituting $U(.) = -(.)^{-1}$, and generalising M to R_1, gives:

v_1:

$$\underset{0 \le a_1 \le R_1}{\text{Max}} \ \underset{K}{E} \Big[-a_1^{-1} + (1+r_1)^{-1} . -(Z(K)(R_1-a_1))^{-1} + (1+r_1)^{-2} . -(K(1-Z(K))(R_1-a_1))^{-1} \Big]$$

$$= \underset{0 \le a_1 \le R_1}{\text{Max}} \ -a_1^{-1} - \underset{K}{E} \Big[(R_1-a_1)^{-1} [(1+r_1)^{-1} (Z(K))^{-1} + (1+r_1)^{-2} (1-Z(K))^{-1} K^{-1}] \Big]$$

$$= \max_{0 \le a_1 \le R_1} -a_1^{-1} - (R_1 - a_1)^{-1} \Big[(1+r_1)^{-1} \mathop{E}_{K} [(Z(K))^{-1} + (1+r_1)^{-1} (1-Z(K))^{-1} K^{-1}] \Big]$$

At optimality $\dfrac{d}{da_1} = 0$ therefore

$$a_1^{-2} = (R_1 - a_1)^{-2} \Big[(1+r_1)^{-1} \mathop{E}_{K} [(Z(K))^{-1} + (1+r_1)^{-1} (1-Z(K))^{-1} K^{-1}] \Big]$$

Expanding Z(k) gives;

$$a_1^{-2} = (R_1 - a_1)^{-2} \Big[(1+r_1)^{-1} \mathop{E}_{K} [1 + (1+r_2)^{-1/2} K^{-1/2} + (1+r_1)^{-1} (1 - (1+(1+r_2)^{-1/2} K^{-1/2})^{-1})^{-1} K^{-1}] \Big]$$

$$= (R_1 - a_1)^{-2} \Big[(1+r_1)^{-1} \mathop{E}_{K} [1 + (1+r_2)^{-1/2} K^{-1/2} + (1+r_1)^{-1} (1 + (1+r_2)^{1/2} K^{1/2}) K^{-1}] \Big]$$

$$= (R_1 - a_1)^{-2} \Big[\mathop{E}_{K} [(1+r_1)^{-2} K^{-1} + (1+r_1)^{-1} + ((1+r_1)^{-1} (1+r_2)^{-1/2} + (1+r_1)^{-2} (1+r_2)^{1/2}) K^{-1/2}] \Big]$$

$$= (R_1 - a_1)^{-2} \Big[(1+r_1)^{-2} \mathop{E}_{K} (K^{-1}) + (1+r_1)^{-1} + ((1+r_1)^{-1} (1+r_2)^{-1/2} + (1+r_1)^{-2} (1+r_2)^{1/2}) \mathop{E}_{K} (K^{-1/2}) \Big]$$

Defining X by $a_1^{-2} = (R_1\text{-}a_1)^{-2}X$

$$\Rightarrow \quad a_1 \quad = \quad (R_1\text{-}a_1)\,X^{-1/2} = (1+X^{-1/2})^{-1}\,X^{-1/2}\,R_1 = (1+X^{1/2})^{-1}R_1$$

$$\Rightarrow \quad a_1^* \quad = \quad (1+X^{1/2})^{-1}\,M$$

As for late resolution, this equation for the early resolution optimum a_1^* allows investigation of the sensitivity of the initial decision to the parameters and in particular to uncertainty.

8.3 Resolution timing sensitivity

In accordance with the theme in this study, that 'solutions' should involve an understanding of the sensitivity of action to belief, the functions providing optimal actions for early and late resolution are now compared:

early: $a_E = (1+X^{1/2})^{-1}\,M$

late: $a_L = (1+W^{1/2})^{-1}\,M$

Now, for the example used,

$$a_E < a_L \text{ if } (1+X^{1/2})^{-1}\,M < (1+W^{1/2})^{-1}\,M,$$

which is true if:

$$X > W,$$

that is if:

$$\Big[(1+r_1)^{-2}\underset{K}{E}[K^{-1}] + (1+r_1)^{-1} + ((1+r_1)^{-1}(1+r_2)^{-1/2} + (1+r_1)^{-2}(1+r_2)^{1/2})\underset{K}{E}[K^{-1/2}]\Big]$$

$$>$$

$$\Big[(1+r_1)^{-2}\underset{K}{E}[K^{-1}] + (1+r_1)^{-1} + ((1+r_1)^{-1}(1+r_2)^{-1/2} + (1+r_1)^{-2}(1+r_2)^{1/2})\big[\underset{K}{E}[K^{-1}]\big]^{1/2}\Big]$$

that is if:

$$\underset{K}{E}[K^{-1/2}] > \big[\underset{K}{E}[K^{-1}]\big]^{1/2}$$

This is true for all bounded distributions on $K > 0$ because then:

$$\frac{1}{+\sqrt{K}} > \frac{1}{K}$$

$$\Rightarrow \quad \underset{K}{E}[K^{-1/2}] > \underset{K}{E}[1/K], \text{ for all distributions, (positive root)}$$

$$\Rightarrow \quad \underset{K}{E}[K^{-1/2}] > \big[\underset{K}{E}[1/K]\big]^{1/2}$$

For this example therefore, the optimal initial use is smaller if the size of K is discovered earlier, and this is true irrespective of the finite discount rate(s) applying to initial or second period choice.

8.4. Discussion

In this case, the early knowledge of K provides an enhanced ability to balance second and third period use. This makes carrying-over resource stocks sufficiently more attractive to outweigh the opposing consideration that, because balancing is improved, any level of satisfaction from later use can be achieved with a smaller stock. Because these opposing considerations seem finely balanced, and the utility function parameter seems intimately involved in establishing the result, it may not generalise to wider contexts.

The result is in general accord with the intuition that earlier resolution is akin to a reduction in uncertainty, and may repay some (opportunity) costs. That is, research may be worthwhile. Here, the early resolution optimum provides the initial decision-maker with a higher objective function value. The increase, over the value of later resolution, is a measure of the value of **partial** information which is analogous to the 'expected value of perfect information' measure widely used in decision theory.

The example demonstrates that the optimal solution depends on the timing of the resolution of uncertainty about the parameters. This is true even though the objective(s) are neutral over the timing of uncertainty resolution in objective values.

The timing of parameter uncertainty resolution would be important in constructing a choice procedure to decide between slowly resolving gambles over the parameters - if this choice procedure was designed to satisfy (or was induced from) more fundamental preferences over 'utility' outcomes. This idea has implications for the examples used in this study. If the objectives over resource use streams are surrogates for objectives on other streams (e.g. consumption), then the former objectives may need to be sensitive to timing of uncertainty resolution.

Objectives which are timing sensitive are easily handled by the recursive approach. In precommitment approaches timing sensitivity generally leads to intertemporal inconsistency, or the problem that the 'optimum' will not be followed hence is not optimal in any sensible way.

The recursive approach avoids this by dealing only with the future possibilities which derive from future choices. The choice procedures involved, whether current, or forecast for the future, can employ timing sensitivities without intertemporal consistency becoming an issue. Investigation of the impact of such sensitivities on justifiable objective forms, and in turn on optimal resource programs, is left to future work.

Chapter Nine

Conclusions

The introduction to this dissertation suggests that it is hard to clearly identify appropriate principles for guiding non-renewable resource use. This is partly because the consequences of non-renewable resource use extend into the distant future and are uncertain, and consequences of this sort are poorly handled in the existing theoretical investigations of decision-making.

The subsequent chapters enlarge on the limitations of the current theoretical approaches, and develop a new recursive approach which better represents the position of decision-making in time. This approach is then used to investigate several issues in non-renewable resource use over time.

This chapter collects the main findings, and discusses their significance in section 9.1. The section clearly identifies what contribution this study makes to the existing knowledge of non-renewable resource decisions, and of the principles guiding them. The study has some implications for policy, and for future research, which are outlined in sections 9.2 and 9.3.

9.1 Main findings

9.1.1 Existing approaches

Decisions will always be made on the basis of principles, or rules of thumb; that is, by using more or less developed guides to action. Principles are normative: they indicate what sorts of actions should be allowed or selected, and so are directed at achieving goals or standards.

Generally, principles are instrumental: they are not goals in their own right, but are likely to help in achieving goals.

Only a fraction of the existing information can ever be brought to bear on a decision. This is because the need for and the nature of decisions cannot be completely foreseen, and the ability to manipulate information is scarce. Seeking further information, before selecting an option, is an option in itself. To be consistent it must be ranked with the other alternatives, and its selection is also governed by principles.

The principles which are relied on change with experience. New principles are sought when there is dissatisfaction with the results of applying existing principles. New principles are also a source of this dissatisfaction; the suggestion is that they would provide better results than existing principles.

Theoretical investigations are a chief source of new principles. A new understanding of the way things operate, and/or of the ways ethical claims may be justified, might reveal that existing principles are defective. Suggestions for modified or altogether new principles might also be found.

Generally, research into principles is motivated by a concern expressed in vague or everyday terms, and the initial task is to clarify this. Some aspects of the concern are inevitably left out in the progression to a concretely defined problem.

Research into principles uses models of the situations in which the principles will be employed. It is useful to view models as being developed in two stages. Firstly, a conceptual framework is adopted. This defines and limits the types of things which can be examined, and how they can be examined. Secondly, the concepts from the framework are used to construct a model. This exactly identifies and explores (a version of) the issue.

The vague concern which motivates this thesis is that people depend on being able to use some things which are irreplaceable. However, the future importance of these things is uncertain. Technological and social changes may lessen (or, for a while, increase) their importance, by changing what substitutes are available, for instance.

The conceptual frameworks of economics can be used to produce models for exploring aspects of this concern. Rewritten in economic terms, the concern is that the material welfare of society depends on a throughput of non-renewable resources or of substitutes for them. Uncertainty about future technological and resource possibilities makes a future collapse in material welfare a possibility. It is not obvious how current principles guiding non-renewable resource use and substitute creation do, can or should account for the future possibilities.

Many well-defined models of aspects of these issues can be constructed. However, the dynamic, uncertain context severely tests the capability of the economic approaches, and the relevance of their results.

The concepts of welfare economics have been used in many investigations involving non-renewable resource owners making price-based decisions under uncertainty. In general, given sufficient competitive pressure, and enough contingent markets or rational expectations, equilibrium prices exist. Further, these are *ex-ante* efficient.

The apparent implication is that the principles for non-renewable resource use do not differ in any marked way from those welfare economics recommends for other resources. In short: allow a decentralised price system to guide the allocation, provided that competitive pressure is maintained, and externalities are treated where this is cost-efficient.

However, this implication is questionable, both because the existence of an equilibrium is sensitive to the underlying assumptions, and because some assumptions are far from 'realistic'. Theoretical explorations show that equilibrium prices can be indeterminate for non-renewable resources, that strategic behaviour is very important in less-than-perfectly competitive situations, and that taxes can prevent efficient outcomes from being attained. Also, the contingent markets and/or expectations which are required to sustain an intertemporal equilibrium do not appear to exist in reality.

The welfare framework itself is questionable. The initial endowment of resources to agents determines which allocation is reached. The **distribution** of welfare at this allocation may be what is most important to most individuals, and may be what is ethically important. However, distributional impacts cannot be determined within the welfare analysis. Neither actions with important distributional implications, nor actions directed at changing the welfare distribution (such as contingent transfers), can be properly explored within the welfare framework.

The welfare approach relies on 'consumer sovereignty' to justify why consumers, as opposed to say voters, should determine what happens to resources. This is a debatable ethical position, and it is more debatable when there is uncertainty, because agents' beliefs about the likelihood of various events, and agents' risk attitudes, then influence the outcome. These beliefs and attitudes are open to interpersonal question in ways that agents' tastes or ethical judgements are not.

Personal beliefs are most critical when there are very few contingent markets, so that unexpressed expectations influence the resource use pattern. Extrapolative expectations are known to be inadequate to sustain an efficient intertemporal equilibrium. There are in reality very few forward and contingent markets for non-renewable resources. This reveals in addition that transaction costs, which are treated as unimportant in this framework, are probably critical: they determine how informed individuals are, and hence what actions are taken.

The welfare economics approach is not properly dynamic. The 'tatonnement' process is assumed to instantaneously bring about *ex-ante* equilibrium plans, and these are imagined to be followed forever after. Agents are implicitly assumed to live for an infinite time and to have intertemporally consistent tastes. They are generally assumed to update their beliefs rationally; i.e. in accordance with Bayes rule.

Optimal growth theory is the other conceptual framework from economics which is widely used in this area: the 'socially optimal' use of non-renewable resources, when there is uncertainty, is explored as an optimisation problem. The optimal aggregate intertemporal pattern for resource use is derived. The theoretical investigations show that this pattern is sensitive to the objective adopted, to the particular uncertainty examined, and to the extent of the uncertainty.

The optimal aggregate resource use patterns have obvious limitations as principles for guiding resource use decisions. One limitation is the very high level of abstraction from the social or institutional structures which are required to bring about any resource use pattern. These structures are in reality only able to produce a limited range of resource use patterns, which may not include the indicated 'optimum': optimization over this smaller range may produce a very different 'optimum'. Further, consideration of the institutional 'costs' may well change the objective, and hence the optimum.

Another limitation of the optimal growth framework is that very few objectives can be explored. The framework produces precommitment solutions, which fix all future actions at the initial time, in accordance with the initial decision-maker's objective. For most objectives, these solutions are intertemporally inconsistent; that is, it is foreseen that future decision-makers will wish to revise the optimal resource use plans, so their optimality is suspect.

Adherence to the precommitment approach is at odds with humans' position **in** time. Intertemporal resource use patterns derived in this way can appear optimal only if it is believed they will be implemented (in aggregate - institutions aside). For this to seem reasonable it must be believed that judgements are unchanging, or at least are intertemporally consistent.

The stationary discounted utilitarian objective is almost universally used, and this is generally adjusted for uncertainty by taking its expected value. For this objective there is always one imaginable sequence of decison-makers for whom the optimal resource use is intertemporally consistent. This sequence, however, amounts to assuming unchanging preferences, which may not be realistic. Also, consistency is achieved at high cost: the objectives or judgements which can be explored cannot reflect much of the richness of attitudes towards an uncertain future. Program risk aversion is sometimes explored, but produces intertemporally inconsistent 'optima'.

The precommitment approach is also conceptually misleading, if optimum resource use comes to be thought of as a whole plan or strategy (for resource use) which is fixed initially and then simply followed. A more adequate 'solution' must allow that the whole strategy is continually updated, and only the initial step of the current strategy is ever expected to be undertaken.

However, the optimal growth type investigations are explicit in relating possible principles for guiding decisions (here, target patterns for resource use) to the ethical positions or preferences which support them.

The welfarist principles implicitly rely on consumer sovereignty for justification, and this does not seem completely morally compelling.

9.1.2 The recursive approach

If choices on non-renewable resource use are to be based on principles arising from an increasingly informed debate, the relations between ethical positions and options for action must be further explored.

One of the difficulties in doing this is that the existing frameworks for examining choices with long-run consequences do not adequately account for the human position in time. Closely related to this is that the implications of many ethical positions, on creating risks for future people, cannot be explored.

In this study the development of a more adequate approach is commenced.

The first step is to generalise the decision-theoretic view of choice. The latter conceptually underpins most optimising choice models, such as consumer choice theory, the theory of the firm, agent-principal theory, multi-attribute utility analysis, and optimal growth theory. Associated modelling/solution techniques, for intertemporal contexts, include decision-trees, the calculus of variations, (stochastic) control theory, and (stochastic) dynamic programming.

In decision-theoretic models the sole decision-maker must, at a given point in time, choose one plan or strategy. These are sequences of future actions, or sequences of functions (on states) which pick out actions. The decision-maker's own criteria are applied in making the selection, and the initial action is undertaken. That is, precommittment solutions are produced.

Unless the decision-maker believes the **whole** plan or strategy will eventually be undertaken, it is not optimal in any sensible way. But it is not possible, at the initial time, to fix later actions. If later actions will be chosen differently, the initial strategy is misleading.

Therefore, the decision-theoretic approach is inadequate when decision criteria may change. It is also inadequate if the criteria are unchanging, but are intertemporally inconsistent. An example of the latter is when every generation applies the same non-exponential discounting procedure. Another example is when the attitudes to risk depend on the delay before risk resolution, relative to the current time.

In this thesis the decision-theoretic model is generalised to cover a sequence of decision-makers. Each undertakes only one initial action. For each initial action there is a forecast covering the future decision-makers, their actions, and the other events. The action-forecast pairs replace strategies as the initial objects of choice. Each forecast is constructed on the assumption that all future decision-makers choose their initial actions, given their forecasts, in the same way.

This multi-stage, recursive approach amounts to forecasting future decisions, conditional on initial actions, rather than selecting future decisions as part of a strategy. The former is a more realistic representation of decision-making at a point in time; there is no overstatement of the initial decision-maker's power to determine the course of events.

Forecast future actions are derived as the outcomes of choices by forecast decision-makers. That is, the initial decision-maker believes that current and forecast decisions involve the same sort of considerations - all decisions are treated consistently. All decisions, in this approach, involve choice between completely specified, slowly resolving, lotteries or gambles.

That is, no decision-maker has available any recourse actions, since future actions are taken by future decision-makers. Clearly, a forecast that (later versions of) the initial decision-maker continues to make the decisions, in a way consistent with that used originally, collapses the recursive approach to the standard decision-theoretic special case.

The recursive approach completely avoids the issue of intertemporal consistency. The action-forecast options never involve later actions which will not be taken, as do inconsistent strategies. The forecast actions are, by construction, explicitly feasible with respect to future decisions, however decision-making procedures are envisaged to evolve. Procedures expressing virtually any time and risk preferences, or ethical positions, can therefore be explored with the approach.

Several modelling structures are compatible with the recursive approach. The structures differ as to whether time is discrete or continuous, and whether actions, exogenous 'random' events, and choice procedures are drawn from discrete or continuous sets.

The recursive structures adopted in this study stay close to the well-known decision-tree model. The time periods are discrete: a decision at the start of the period results in an action being undertaken; this is closely followed by resolution of some uncertainty when a subjectively random event occurs; the action and event bring about a state of affairs, or outcome, which prevails until the end of the period; the uncertainty about the next choice procedure is resolved just before the next period commences.

This specification is pessimistically biased in that each decision must precede resolution of the whole of the 'periods-worth' of uncertainty. Reversing the order of actions and events at the beginning of the period reverses the bias. The discrete time framework cannot fully reflect the gradual resolution of uncertainty and adjustment of action which occurs in reality.

The specification is completed by adopting Bayesian rationality as a standard applying to each decision-maker's beliefs about future occurrences, and requiring that every decision-maker has the **same** beliefs, appropriately conditioned to reflect the preceding actions and events.

This stringent requirement conflicts with the observation that beliefs can and do change in ways not captured by updating prior to posterior beliefs in Bayesian fashion. However, if this sort of change (say, a drift from one Bayesian rational structure to another) is foreseeable, it must be incorporated as a possibility in the preceding beliefs structures, which preserves the assumption of Bayesian consistency within and between beliefs. If the change is not foreseeable then it is difficult to see how it can sensibly be allowed for beforehand. This is an unavoidable limitation of 'rational' forecasting: it is rational to believe that new learning will revise existing forecasts in unforeseen ways.

Accordingly, it is rational for uncertainties about some events to be inexpressible as single (subjective) probability judgements. This uncertainty may be represented by a **set** of Bayesian rational initial belief structures, between which the initial decision-maker cannot judge. The sensitivity of the optimal commitment to a likelihood judgement over this set can then be explored. Alternatively the decision-maker may wish to invoke the principle of insufficient reason and attribute uniform subjective likelihood to the set.

Given this full specification of the **content** of a belief structure, the existence of a corresponding optimal action-forecast can be investigated. When actions, events, and choice procedures fall into finite sets and choice procedures are well defined it is sufficient for existence of an optimum that there is a date at which each preceding decision-makers' concerns, for occurrences after that date, can be represented as a function of the state on that date. This condition is sufficient to allow a backwards recursion technique to iteratively identify each preceding period's optimum, and eventually the initial optimum.

This condition is satisfied if there is an horizon which is agreed to by all preceding decision-makers (according to initial belief). In the generalised framework this requires agreement of the decision-makers **immediately** preceding the time. An alternative sufficient condition is that there is a time after which, for each state, the succeeding decision-making procedures are intertemporally consistent (according to initial belief), and allow solutions for all succeeding states to be found. If no optimum exists then there are no justifiable actions corresponding to that belief set. Such cases provide an interesting commentary on what is required to justify action.

The recursive framework created by generalising the decision-theoretic view therefore can both formulate and investigate a number of previously unexplored issues. Among other things, the recursive framework can be used to extend the mapping from ethical positions to optimal initial actions vis a vis non-renewable resources. Principles guiding resource use can then be developed in a more informed way.

The recursive structure of the framework makes it well suited for numerical approximation where analytical methods fail. The belief-optimum mapping could be built-up piecewise, from the solutions to numerical formulations of cases. When all choice procedures are always backwardly recursive an algorithm akin to dynamic programming can be exploited in finding the optimal action-forecast.

When used analytically, the framework provides new results on optimal resource use patterns. These turn out to be sensitive to changing choice procedures, to the uncertainty about choice procedures, and to the timing of the resolution of uncertainty.

The sequence of choice procedures investigated allows the discount rate adopted by each period's decision-maker to vary. This might approximate changes to the position adopted on intergenerational equity, with a higher discount rate indicating less regard for the future. Alternatively, a higher discount rate could reflect a higher probability being attributed to the possibility of 'Armageddon'.

The optimal initial resource use is lower when future discount rates are higher or lower than current rates, for the formulation studied. Therefore, ignoring the possibility of a drift in discount rates systematically biases resource use patterns towards the present.

All precommitment investigations to date ignore the possibility that discount rates change with time, which suggests that the collection of precommitment results may be misleading, in that they do not carry over when the initial decision-maker cannot choose later actions. It is therefore important that in future work this effect is investigated for other assumptions on the utility function and the horizon, and for more general patterns of fluctuation in the discount rate.

When future discount rates are in addition uncertain, a program risk-neutral initial decision-maker reduces initial resource use by comparison with the certain mean case, for the formulations investigated, and discount rates in the usual range. The bias in the precommitment results is therefore reinforced, and not offset by allowing for uncertainty. This in turn reinforces the need to investigate the bias in wider contexts, so that the precommitment results can be better judged.

In investigating the importance of the timing of uncertainty resolution, the recursive approach is able to clearly separate the effects of better information from the influence of a 'desire to know'. For the formulation used, early resolution provides the initial decision-maker with an increase in the expected value of later actions, so that optimal initial resource use is reduced. Research to bring about early resolution can outweigh some (opportunity) costs. The effect, however, seems finely balanced and may not extend to other formulations.

The analytical results demonstrate that the framework is sufficiently tractable to provide new insights. The iterative analytical procedure takes advantage of the temporally separable objectives. The same advantage is available if the objectives are separable by sub-tree only, which permits a wide range of attitudes to uncertainty to be investigated. Therefore, more general formulations should not present formidable analytical difficulties.

9.2 Implications for policy

The cluster of principles which guides decisions is termed 'policy', whether or not it is formally set down. Most of the findings discussed above have at least indirect implications for non-renewable resource policy, because they identify the features required for a sound investigation of decisions with long-term, uncertain consequences. These features are also relevant to many other policy spheres. The most direct implications for resource policy, and for policy in general, are outlined in this section.

The use of existing theory

There are reasons for caution in using the existing theoretical investigations as a basis for policy on non-renewable resource use. In both the descriptive and prescriptive literatures the high level of abstraction from the incentives and opportunities individuals face, and from 'realistic' technological and environmental conditions, is a serious weakness.

Also, the recommendatory power of the analyses is hard to gauge, because their ethical foundations are generally implicit and open to several interpretations. However, this thesis concentrates on the weaknesses in the theoretical treatment of dynamics and uncertainty.

Policy directed at *ex-ante* efficiency (Pareto Optimality) must consider extra factors when there is significant uncertainty. This criterion takes as given whatever (probabilistic) opportunities and incentives are perceived by individual economic agents. If individuals are poorly informed then they will make poor decisions, but this is not inefficient. In addition, the 'free-rider' problem often occurs in the production of information, and counter-measures do not in general restore efficiency.

Therefore, policy on how informed to be, or on the provision of information to individuals, cannot be based purely on efficiency considerations. Other measures of merit must be applied to the activities which generate and disseminate information. Such activities include trading in futures markets, resource exploration, research into substitute possibilities, and general education.

There is a further reason for questioning the relevance of *ex-ante* efficiency to policy, when there is uncertainty. The *ex-ante* opportunities may be less ethically justifiable, as a measure of merit, than the range of *ex-post* outcomes. If so, observable market prices (which reflect individuals' *ex-ante* judgements) are not good measures of merit. Policies (such as 'allocate resources so as to maximize their expected net present value') which use these prices are therefore questionable.

Caution is also required in using the existing prescriptive Utilitarian or Rawlsian resource investigations as a basis for policy. Among other things, the treatment of dynamics and uncertainty in these investigations has serious limitations. The investigations poorly reflect the position of decision-making in time, overstate the decision-maker's power to determine future events, and deal with few of the possible attitudes towards uncertain future outcomes. Investigations using the more adequate recursive approach show that the 'optimal' resource use patterns are sensitive to these assumptions. Therefore, the existing prescriptive literature may be a misleading guide for policy.

Many intertemporal planning exercises are questionable on both the grounds developed above: they measure merits by observed and forecast *ex-ante* prices, or by the consumer and producer surplus existing at those prices; they seek an 'optimal' intertemporal plan or strategy by maximising a precommitment objective, without explicitly considering whether this will be followed by future decision-makers.

Implications of the recursive approach

Future decision-makers must be considered in analysis of public policy initiatives. Both 'public sentiment' and governments change over time, and the public choice procedures and 'preferences' are likely to change accordingly. This likely change should be explicitly considered in policy development.

Many policy initiatives involve transition costs, and are (from the initial viewpoint) worth implementing only if they are continued for a reasonable time. The method of evaluating such initiatives must have a way of giving them little merit if their continuation by future governments is unlikely.

Other initiatives are only worth implementing if they will be discontinued when things go wrong. The method of evaluating these initiatives must have a way of giving them little merit if their discontinuation by future decision-makers is unlikely, even though things have gone wrong. An instance where this is important is where continuation occurs because the choice procedure of the existing government changes, to reflect its desire not to 'lose face' by reversing course.

The interaction between public sentiment and public choice procedures is important in gauging the merits of different political systems, and of the policies employed in them. Sentiments may cycle through time, (say) from preferring centralized allocation to preferring decentralized allocation. If the cyclical nature of sentiment is not recognised then radical policy changes may be continually undertaken, and large transition costs may be continually borne.

In the extreme, failure to recognise cycles might lead to destabilising public policy changes which cause increasing swings in sentiment. Alternatively, policy changes may be self-reinforcing.

The importance of these interactions cannot be examined with the existing precommitment approaches to choice. The recursive approach to intertemporal decision-making clearly identifies why and how policy evaluation routines should allow for the possibility of changes in, and interaction between, policy and public sentiment. The recursive approach may therefore both motivate and underpin the construction of improved principles to guide decisions.

9.3 Implications for research

The chapters above have touched on many diverse areas where research could improve understanding of principles for non-renewable resource use. Many of these, such as institutions for guiding resource use, are of general importance, but are discussed below only for the narrower range of issues investigated in this study. The research implications of the original problem analysis are covered first. Further research into the recursive approach itself is then briefly discussed before applied research employing recursive approaches is outlined.

9.3.1 Existing approaches

Justifiable principles for guiding non-renewable resource decisions must be based on knowledge of both the consequences of the principles and their merits. Currently, the assumptions used in the descriptive or predictive investigations are very different from those used in the prescriptive or normative investigations. The former in general employ more 'realistic' assumptions about ownership patterns and technology, while the latter in general are more explicit about the ethical basis of their measures of merit.

While neither set of assumptions is particularly convincing as a representation of the real situation, the recommendatory power of the analyses is reduced by the large difference in assumptions. Improved recommendatory power would be possible if each type of investigation employed similar (sets of) assumptions. Therefore, descriptive research, which continues to analyse the performance of resource ownership and technological situations, but which uses wide ranging measures of merit, would be valuable. Equally valuable would be prescriptive research which used more realistic assumptions on resource ownership and technology.

The ways in which people form opinions (or remain uninformed), about the uncertain distant future, are poorly understood. The mechanisms here are crucial to the resource use patterns which eventuate in both centralized and decentralized societies. Empirical and theoretical research into expectations formation in various institutional settings might greatly improve the 'realism' of the assumptions.

It is not clear why resources are currently priced in only a small number of futures and contingent markets. These markets are a potentially important forum in which long run expectations can be expressed and evolve, and so influence resource use. The nature of the transaction costs, and the reactions to uncertainty, which prevent the operation of long run futures markets, are worthy of further empirical and theoretical research.

The range of mechanisms for guiding long term resource use has not been widely explored. Particularly deserving of research are ways of fostering information generation and exchange. Among these is the possibility of supplementing market mechanisms with centrally provided information or education. These are important because of the key role of expectations in determining the long run resource allocation, and because the 'free-rider' problems with information are well-known.

Measures of the merits of uncertain (including probabilistic) sets of intertemporal outcomes are not well developed. Empirical and axiomatic research is necessary to discover the ways individuals might value such prospects, if predictive models of resource use are to be improved.

Similarly, measures of the (social) merits of allocation systems, and of their consequences, are not well developed for uncertain dynamic contexts. Research into the differences between sequences of *ex-ante* and *ex-post* allocations would improve the basis for judging their ethical merits.

'Flexibility', 'resilience', and 'robustness' are often mentioned as being desirable attributes for resource allocation mechanisms, or for social systems in general. Research which explores measurable definitions of these attributes, and investigates how they are furthered by various principles for guiding resource decisions, would allow the wider merits of resource use arrangements to be gauged.

9.3.2 The recursive approach

Many aspects of the recursive approach require further investigation. Choice procedures on probabilistic trees are central to the approach but are not well-explored. The procedures individuals use, or at least conform with, must be further investigated empirically. The choice procedures which are compatible with various sets of 'rationality' axioms require research.

A wide range of attitudes to uncertainty seem reasonable as choice procedures within a recursive model. An investigation of the dynamics of choice procedures, in interaction with the consequences they bring about, might suggest that some forms are self-reinforcing over time. Choice procedures are therefore likely to evolve towards these forms, and this knowledge would provide for better predictive theories.

Choice procedure forms which can be specialised to represent arbitrary attitudes to uncertain future outcomes, while remaining tractable analytical tools, are required if the recursive approach is to be widely used. Research which builds on similar investigations of precommitment objectives is needed to establish these forms.

Recursive structures which employ continuous action and event spaces, and continuous time, require investigation. This is necessary to enable a wider comparison of results with those of precommitment analyses, which generally use continuous spaces. Many powerful analytical tools, differential calculus in particular, might be brought to bear in recursive optimisation over continuous spaces, as the special cases of Chapters Six to Eight demonstrate.

The question of existence of an optimal commitment-forecast requires further treatment, both for the discrete case examined here and for continuous cases. Exact necessary and sufficient conditions for the existence of an optimum would delineate the cases which are open to treatment in recursive structures.

The conditions would also indicate which belief sets have associated optimal initial actions, and which have not. This might allow a broader discussion of the rationality of beliefs, in terms of which beliefs help in selection of action, and which do not.

Research into the topological nature of recursive structures seems likely to be fruitful: the optimal commitment-forecast is analogous to an equilibrium within the space of beliefs, so the fixed point properties of the space may be useful in establishing when an optimum exists.

Comparisons of the recursive structures with game theory models may be similarly enlightening: the commitment-forecast optimum is analagous to a collection of Stackelberg optima, with each envisaged decision-maker using the choices of all future decision-makers in forming a reaction function.

The strong assumption that beliefs conform to a single Bayesian rational structure requires further research. The assumption may be able to be weakened, without preventing existence of an optimal commitment. Vagueness might be allowed for by requiring that the system transitions be specified only up to membership of sets of states, and these could become 'vaguer' with time. The relationship between the horizon setting conditions, and growing vagueness, also needs further inquiry.

The recursive approach can be applied in all situations previously treated with precommitment formulations, and may add to or detract from confidence in the precommitment results. A full research program employing recursive structures is required, so as to further test the capability of the approach and to build a set of recursive results of comparable breadth to the precommitment results. A true comparison of the gain in understanding afforded by the recursive approach can only then be made.

Within this program there are two priority research topics. The first is to check for systematic biases in precommitment results, which might follow from the overstatement of the powers of the initial decision-maker. Preliminary evidence of this is found in Chapter 6 above, but a more wide-ranging examination, which relates the results to the assumed parameters, is required.

Exposure of a systematic bias would allow the precommitment results to be heuristically adjusted, until such time as more complete results are available. Exposure of a systematic bias would also motivate the more complete investigation.

The second topic with high priority is to investigate the implications of choice procedures incorporating attitudes to uncertainty which are outside the scope of precommitment approaches. Important among these are choice procedures which separate risk attitudes from 'time preference'.

In the standard utilitarian objective (the maximization of the expected value of the intertemporal integral of discounted utility) the implicit risk attitudes are affected by the discounting procedure. As a consequence, all investigations using this objective implicitly assume decreasing risk aversion as the risk becomes more remote in time. Choice procedures which reflect the 'opposite' tendency, by exhibiting increasing risk aversion as the date of the risk becomes more remote, seem worthy of early investigation.

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Appendix

Methods for Uncertainty

A.1 Expected utility and risk attitudes

Expected utility maximization is illustrated in Figure A.1 for a 50/50 gamble between two outcomes (of income, perhaps) with the numerical values 1 and 3 respectively. The discussion applies to evaluating any action with an uncertain outcome. The indicator is monotonically increasing, so more (income) is better, and the shape of the indicator captures the 'attitude to risk'.

The expected numerical value of the gamble is 2, which, if received with certainty, has utility G. The expected utility of the gamble is H, less than G, so the agent is 'risk-averse' towards this gamble. Letting E represent the expectation operator,

$$G = U(E[i]) > E[U(i)] = H$$

The 'certainty-equivalent' of the gamble is J. This certain consequence is of equivalent worth to the gamble. Another indication of risk-aversion is that J is less than 2. Gambles over the range 0-1 in Figure A.1 have an expected utility greater than the utility of the expected value of the gamble - the agent is 'risk-loving' over this range.

Local convexity of the utility indicator implies local risk-loving, local concavity implies local risk-aversion, and local linearity implies local risk-neutrality. Two measures of risk attitude are (Pratt, 1964):

the 'index of absolute risk aversion', $r(i) = -U''(i)/U'(i)$,

and the 'index of relative risk-aversion', $R(i) = i\,r(i)$.

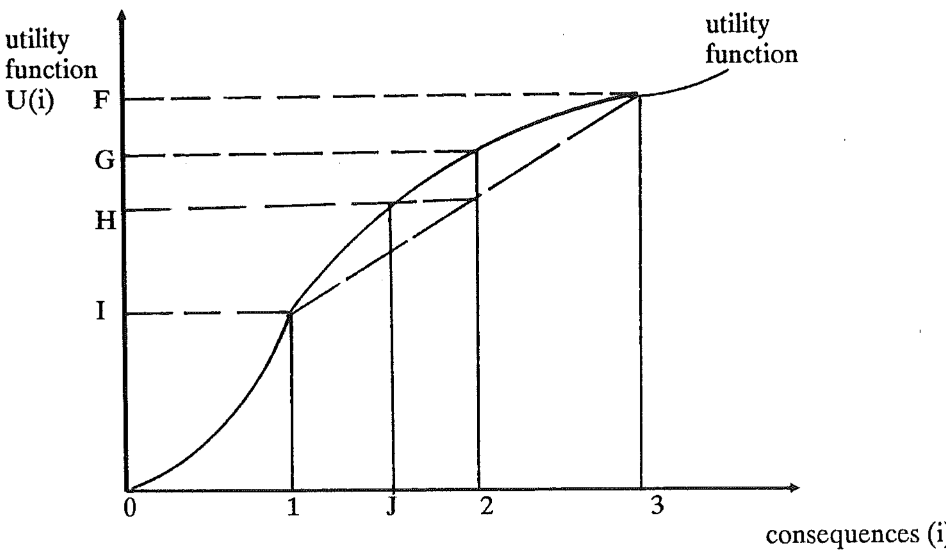

Figure A.1 Expected utility maximization

Both measures are zero under risk-neutrality, and agent A is locally more risk averse than agent B if $r_A(i) \geq r_B(i)$. The class of 'constant-relative-risk-aversion utility indicators' is often used; these functions exhibit constant elasticity of marginal utility if applied to certain consequences.

A.2 Mean-preserving increases

Comparisons of the amount of uncertainty in various situations use the notion of a 'mean-preserving-spread' rather than the variance or entropy of probability distributions. This is so that the mean can be held constant and changes in uncertainty alone can be investigated. As depicted in the shift from distribution A to distribution B in Figure A.2, for symmetric distributions with finite range one mean-preserving-spread is a shift to a new symmetric distribution with the same expected value but a wider range of possibilities (Rothschild and Stiglitz, 1970, 1971; Diamond and Stiglitz, 1974).

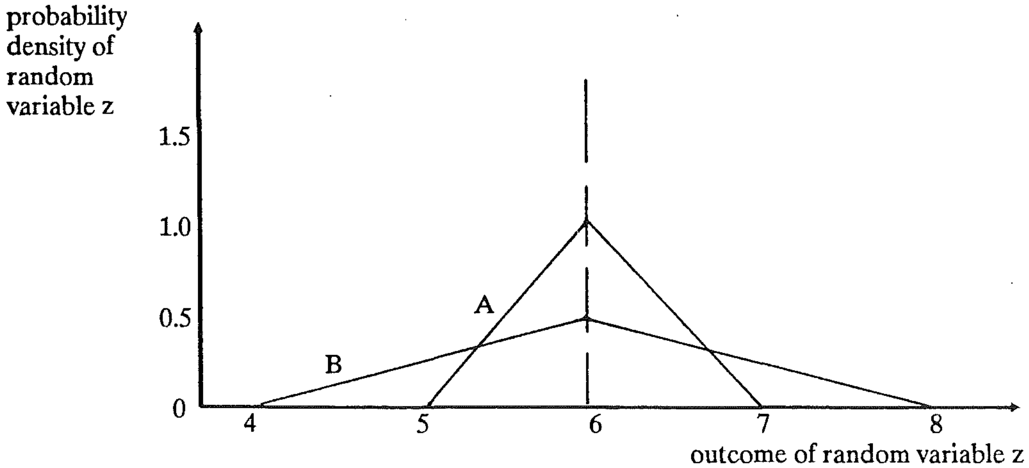

Figure A.2: Mean-preserving-increase in uncertainty

A .3 Jensen's Inequality

Jensen's Inequality is often used in investigating the impact of increasing uncertainty in simple situations (Lippman and McCall, 1981). For any random variable z and function Y:

$$E[Y(z)] \quad \begin{matrix} \geq \\ = \\ \leq \end{matrix} \quad Y(E[z]) \text{ as } Y \text{ is } \begin{matrix} \text{convex} \\ \text{linear} \\ \text{concave} \end{matrix} \quad \text{in } z.$$

Economic investigations often examine how the expected value of some function Y changes as uncertainty about some variable z increases. This is illustrated in Figure A.3: in each part the probability density function (pdf) over z (in the lower right quadrant) is given, and the relationship

Y(z) is known, so the pdf over Y (in the upper left quadrant) can be derived.

In Figure A.3a certainty that z takes the central value gives Y a value of A, and a mean-preserving increase in uncertainty about z increases the expected value of Y from A to B, because Y(z) is convex in z. In Figure A.3b the increase in uncertainty lowers the expected value because Y(z) is concave in z.

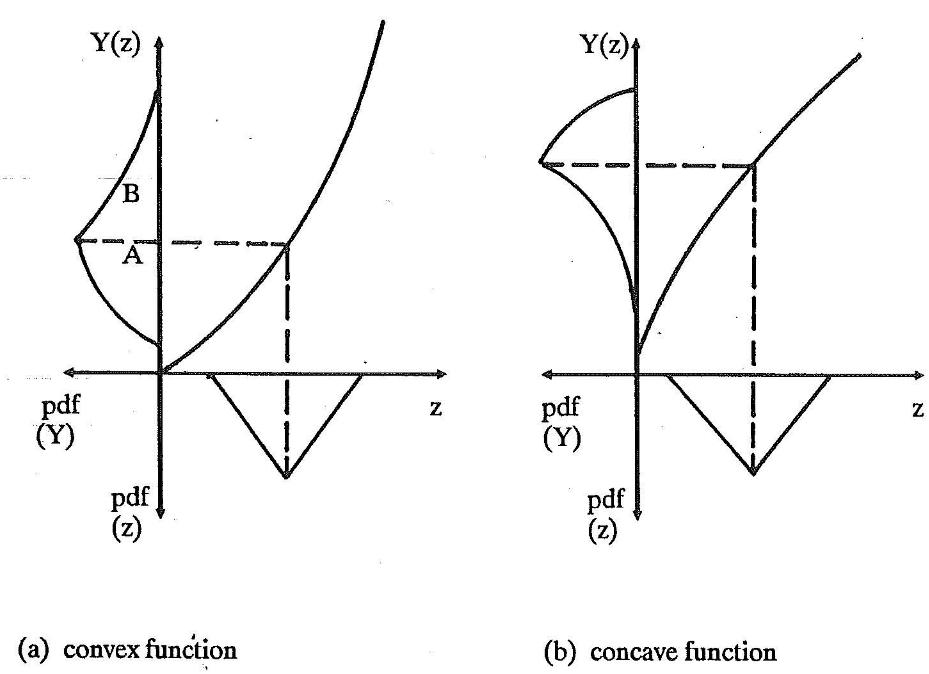

Figure A.3: Jensen's Inequality and increasing uncertainty